

Lecture 15: Feasible GLS; summary of key formulas

Econometrics ■ Unit I: Introduction and generalized least squares

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B.Sc. (Hons) Statistics, Semester V
Academic Session 2026–2027

Previous lecture

Lecture 14: A worked example of GLS.

Today

- When Ω is unknown: feasible GLS
- Summary of key formulas

In practice Ω is rarely known. Usually it depends on a few unknown parameters, for example the correlation ρ in Chapter 5 or group variances in Chapter 4. The *feasible GLS* (FGLS) procedure is:

1. Estimate the model by OLS and obtain the residuals.
2. Use the residuals to estimate the unknown parameters of Ω , giving $\hat{\Omega}$.
3. Compute $\hat{\beta}_F = (X'\hat{\Omega}^{-1}X)^{-1}X'\hat{\Omega}^{-1}y$.

- Because $\hat{\Omega}$ is calculated from the data, $\hat{\beta}_F$ is no longer an exactly linear function of y , and Aitken's theorem does not apply to it exactly.
- In large samples, if $\hat{\Omega}$ is a good estimate, FGLS behaves like GLS.
- Chapters 4 and 5 give the FGLS procedures for heteroskedasticity and autocorrelation.

- Model: $y = X\beta + \varepsilon$, $E(\varepsilon) = 0$, $\text{Var}(\varepsilon) = \sigma^2\Omega$, Ω known and positive definite.
- OLS: unbiased, but

$$\begin{aligned}\text{Var}(b) &= \sigma^2(X'X)^{-1}X'\Omega X(X'X)^{-1}, \\ E(s^2) &= \sigma^2 \text{tr}(M\Omega)/(n - k).\end{aligned}$$

- Transformation: $T\Omega T' = I$, $T'T = \Omega^{-1}$; $y^* = Ty$, $X^* = TX$, $\text{Var}(T\varepsilon) = \sigma^2 I$.

- GLS (Aitken) estimator: $\hat{\beta}_G = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y$, minimising $(y - Xb)'\Omega^{-1}(y - Xb)$.
- $E(\hat{\beta}_G) = \beta$, $\text{Var}(\hat{\beta}_G) = \sigma^2(X'\Omega^{-1}X)^{-1}$, $s_G^2 = e'_G\Omega^{-1}e_G/(n - k)$.
- Aitken's theorem: $\text{Var}(\tilde{\beta}) = \text{Var}(\hat{\beta}_G) + \sigma^2D\Omega D'$ for any linear unbiased $\tilde{\beta}$; GLS is BLUE.
- FGLS: replace Ω by an estimate $\hat{\Omega}$.

Practice

- Exercise 2.7 (Exercises and Solutions booklet).
- R practical P04: Feasible GLS for annual Nile river flow.

Next lecture

Lecture 16 begins Unit II: Multicollinearity and heteroskedasticity.