

Lecture 29: The Goldfeld–Quandt test; comparison of the three tests

Econometrics ■ Unit II: Multicollinearity and heteroskedasticity

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Previous lecture

Lecture 28: The Breusch–Pagan test.

Today

- Goldfeld–Quandt test
 - Setting and procedure
 - Why F_{GQ} has an F distribution
- Comparison of the three tests

This test (Goldfeld and Quandt, 1965) is used when the error variance is suspected to increase (or decrease) with one variable z , often one of the regressors. For example, $\sigma_i^2 = \sigma^2 x_i^2$.

1. Arrange the observations in increasing order of z (move each complete row: y and all regressors together).
2. Omit c central observations and divide the rest into a low group of n_L and a high group of n_H observations. Each group must have more than k observations.
3. Fit the regression *separately* to each group. Let RSS_L, RSS_H be the residual sums of squares, with degrees of freedom $\nu_L = n_L - k$ and $\nu_H = n_H - k$.

4. Compute

$$F_{GQ} = \frac{\text{RSS}_H / \nu_H}{\text{RSS}_L / \nu_L}.$$

5. Under H_0 , $F_{GQ} \sim F_{\nu_H, \nu_L}$. Reject H_0 (in favour of variance increasing with z) if $F_{GQ} > F_{\alpha; \nu_H, \nu_L}$.
- If the two groups have equal size, $\nu_H = \nu_L$ and $F_{GQ} = \text{RSS}_H / \text{RSS}_L$.
 - If the variance is suspected to *decrease* with z , use the reciprocal ratio.

Why F_{GQ} has an F distribution

Assume normal, independent errors with common variance σ^2 under H_0 .

Step 1. Each group is a separate regression satisfying the classical assumptions, so

$$U_H = \frac{\text{RSS}_H}{\sigma^2} \sim \chi_{\nu_H}^2, \quad U_L = \frac{\text{RSS}_L}{\sigma^2} \sim \chi_{\nu_L}^2.$$

Step 2. U_H and U_L are independent because they come from different, independent observations.

Step 3. By definition of the F distribution, the ratio of two independent chi-squares each divided by its degrees of freedom is F :

$$\frac{U_H/\nu_H}{U_L/\nu_L} = \frac{\text{RSS}_H/(\sigma^2\nu_H)}{\text{RSS}_L/(\sigma^2\nu_L)} = \frac{\text{RSS}_H/\nu_H}{\text{RSS}_L/\nu_L} = F_{GQ} \sim F_{\nu_H, \nu_L}.$$

The unknown σ^2 cancels. Under H_1 (variance increasing with z), RSS_H tends to be large, so large values of F_{GQ} lead to rejection.

How many central observations to omit?

- There is no fixed rule.
- The central observations are omitted to make the difference between the two groups sharper, but omitting too many leaves fewer degrees of freedom and makes the test weak.
- Many textbooks take c to be about one-quarter of n (for example, $c = 8$ when $n = 30$, and $c = 16$ when $n = 60$); some authors suggest about one-third of n .
- In every case each group must keep more than k observations.
- The test is not effective if the variance does not change monotonically with z .

Example 4.3 (Goldfeld–Quandt)

There are $n = 30$ observations. After ordering by x , omit $c = 6$ central observations, leaving 12 in each group. A model with intercept and one slope is fitted, so $k = 2$ and $\nu_L = \nu_H = 10$. Suppose $\text{RSS}_L = 10$ and $\text{RSS}_H = 50$. Then

$$F_{GQ} = \frac{50/10}{10/10} = 5.$$

Since $F_{0.05;10,10} = 2.9782$ and $5 > 2.9782$, reject homoskedasticity in favour of variance increasing with x ($p\text{-value} \approx 0.009$).

Exam tip

For every test write four things: the null hypothesis, the statistic, its distribution under H_0 , and the decision rule. Remember that only the Goldfeld–Quandt test is exact; the other two are approximate.

Comparison of the three tests

	Bartlett	Breusch–Pagan	Goldfeld–Quandt
Used when	Data in independent groups	Variance may depend on known variables z	Variance may increase with one variable
Statistic	B in (4.1)	$\Theta = \text{ESS}/2$	F_{GQ}
Distribution under H_0	Approx. χ_{m-1}^2	Approx. χ_{p-1}^2	Exact F_{ν_H, ν_L}
Main caution	Sensitive to non-normality	Detects only patterns linked to z	Needs correct ordering variable

Practice

- Exercise 4.5 (Exercises and Solutions booklet).
- R practical P07: Diagnosing heteroskedasticity in house prices.

Next lecture

Lecture 30: Estimation under heteroskedasticity: WLS, transforming the model, feasible GLS.