

Lecture 33: Consequences of autocorrelated disturbances

Econometrics ■ Unit III: Autocorrelation and instrumental variables

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Previous lecture

Lecture 32: Moments of the AR(1) disturbances.

Today

- Consequences of autocorrelated disturbances

(1) OLS remains unbiased. As before,

$$E(b) = \beta + (X'X)^{-1}X'E(u) = \beta.$$

(2) The variance of OLS changes. With $\text{Var}(u) = \sigma_v^2\Omega$,

$$\text{Var}(b) = \sigma_v^2(X'X)^{-1}X'\Omega X(X'X)^{-1} \neq \sigma_u^2(X'X)^{-1}.$$

Take $y_t = \beta x_t + u_t$ (no intercept). Then

$$b = \frac{\sum x_t y_t}{\sum x_t^2} = \beta + \frac{\sum x_t u_t}{\sum x_t^2},$$

and

$$\begin{aligned}\text{Var}(b) &= \frac{1}{(\sum x_t^2)^2} \text{Var}\left(\sum_t x_t u_t\right) \\ &= \frac{1}{(\sum x_t^2)^2} \left\{ \sum_t x_t^2 \sigma_u^2 + 2 \sum_{s < t} x_t x_s \rho^{t-s} \sigma_u^2 \right\} \\ &= \frac{\sigma_u^2}{\sum x_t^2} \left\{ 1 + 2\rho \frac{\sum_t x_t x_{t-1}}{\sum x_t^2} + 2\rho^2 \frac{\sum_t x_t x_{t-2}}{\sum x_t^2} + \dots \right\}.\end{aligned}$$

The usual formula gives only the first term $\sigma_u^2 / \sum x_t^2$. In economics both ρ and the correlation of x_t with x_{t-1} are usually positive, so the bracket exceeds 1 and *the usual formula underestimates the true variance*.

For $y_t = \mu + u_t$, OLS gives $\bar{y}$ and

$$\text{Var}(\bar{y}) = \frac{\sigma_u^2}{n^2} \left\{ n + 2 \sum_{h=1}^{n-1} (n-h)\rho^h \right\},$$

since there are $n-h$ pairs of observations h periods apart.

For $n = 4$, $\rho = 0.5$, $\sigma_u^2 = 1$:

$$\text{Var}(\bar{y}) = \frac{4 + 2\{3(0.5) + 2(0.25) + 1(0.125)\}}{16} = \frac{4 + 4.25}{16} = 0.5156,$$

more than double the value $1/4 = 0.25$ under independence.

(3) s^2 is biased (usually downward when $\rho > 0$), because

$$E(e'e) = \sigma_v^2 \text{tr}(M\Omega) \neq \sigma_u^2(n - k).$$

(4) Tests are invalid.

- The usual t and F tests and confidence intervals are unreliable.
- With positive autocorrelation, standard errors are typically understated, t ratios overstated, and variables may appear significant when they are not.
- R^2 may also be overstated.

(5) OLS is not efficient. By Aitken's theorem, GLS is BLUE and OLS is not.

Practice

Re-work the derivations and examples of this lecture without looking at the slides.

Next lecture

Lecture 34: Detection: graphical method; the Durbin–Watson statistic and its relation with ρ .