

Lecture 34: Detection: graphical method; the
Durbin–Watson statistic and its relation with ρ
Econometrics ■ Unit III: Autocorrelation and instrumental variables

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Previous lecture

Lecture 33: Consequences of autocorrelated disturbances.

Today

- Detection of autocorrelation
 - Graphical method
 - The Durbin–Watson test

Plot the OLS residuals e_t against time, or e_t against e_{t-1} .

- Long runs of residuals of the same sign, or points concentrated in the first and third quadrants of the (e_{t-1}, e_t) plot, suggest positive autocorrelation.
- Rapid alternation of signs suggests negative autocorrelation.

The Durbin–Watson test

The test was proposed by Durbin and Watson (1950, 1951).

Assumptions.

- (i) The model has an intercept.
- (ii) The regressors are non-stochastic; in particular, the lagged dependent variable y_{t-1} is not a regressor.
- (iii) The disturbances follow AR(1) and are normal.
- (iv) There are no missing observations.

Hypotheses. $H_0 : \rho = 0$ against $H_1 : \rho > 0$ (positive autocorrelation), or against $H_1 : \rho < 0$.

Statistic. From the OLS residuals,

$$d = \frac{\sum_{t=2}^n (e_t - e_{t-1})^2}{\sum_{t=1}^n e_t^2}.$$

Step 1 (expand the numerator).

$$\sum_{t=2}^n (e_t - e_{t-1})^2 = \sum_{t=2}^n e_t^2 + \sum_{t=2}^n e_{t-1}^2 - 2 \sum_{t=2}^n e_t e_{t-1}.$$

Step 2 (approximate). For large n , $\sum_{t=2}^n e_t^2 \approx \sum_{t=1}^n e_t^2$ and $\sum_{t=2}^n e_{t-1}^2 \approx \sum_{t=1}^n e_t^2$, since each differs from the full sum by only one term. Hence

$$d \approx \frac{2 \sum e_t^2 - 2 \sum e_t e_{t-1}}{\sum e_t^2} = 2 \left(1 - \frac{\sum_{t=2}^n e_t e_{t-1}}{\sum_{t=1}^n e_t^2} \right) = 2(1 - \hat{\rho}),$$

where $\hat{\rho} = \sum e_t e_{t-1} / \sum e_t^2$ is the sample first-order autocorrelation of the residuals.

Step 3 (range). Since $-1 \leq \hat{\rho} \leq 1$, we get $0 \leq d \leq 4$ approximately. (In fact $0 \leq d \leq 4$ exactly, because $(e_t - e_{t-1})^2 \leq 2e_t^2 + 2e_{t-1}^2$.)

- $\hat{\rho} = 0 \Rightarrow d \approx 2$: no autocorrelation.
- $\hat{\rho} = +1 \Rightarrow d \approx 0$: strong positive autocorrelation.
- $\hat{\rho} = -1 \Rightarrow d \approx 4$: strong negative autocorrelation.

The approximation also gives a quick estimate

$$\hat{\rho} \approx 1 - d/2.$$

Practice

- Exercise 5.3 (Exercises and Solutions booklet).

Next lecture

Lecture 35: The Durbin–Watson test: decision rule, examples and limitations.