

Lecture 35: The Durbin–Watson test: decision rule, examples and limitations

Econometrics ■ Unit III: Autocorrelation and instrumental variables

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Previous lecture

Lecture 34: Detection: graphical method; the Durbin–Watson statistic and its relation with ρ .

Today

- Detection of autocorrelation
 - The Durbin–Watson test

The exact distribution of d depends on X , so one table cannot cover every data set. Durbin and Watson therefore tabulated a lower bound d_L and an upper bound d_U . These depend only on n , on the number of regressors k' (not counting the intercept), and on α .

For a test against positive autocorrelation:

Value of d	Decision
$d < d_L$	Reject H_0 : evidence of positive autocorrelation
$d_L \leq d \leq d_U$	Inconclusive
$d > d_U$	Do not reject H_0

For a test against negative autocorrelation, apply the same rule to $4 - d$: reject if $4 - d < d_L$, that is, $d > 4 - d_L$.

Combining both sides:

Region	Conclusion
$0 \leq d < d_L$	Positive autocorrelation
$d_L \leq d \leq d_U$	Inconclusive
$d_U < d < 4 - d_U$	No autocorrelation
$4 - d_U \leq d \leq 4 - d_L$	Inconclusive
$4 - d_L < d \leq 4$	Negative autocorrelation

Example 5.2 (Computing d)

An intercept-only fit gives the residuals

$$e = (2, 1, 0, -1, -2, -1, 0, 1)'.$$

- $\sum e_t^2 = 4 + 1 + 0 + 1 + 4 + 1 + 0 + 1 = 12$.
- Differences $e_t - e_{t-1}$: $-1, -1, -1, -1, 1, 1, 1$; their squares sum to 7.

So $d = 7/12 = 0.583$ and $\hat{\rho} \approx 1 - 0.583/2 = 0.71$: strong positive autocorrelation is indicated. The formal decision uses the table values of d_L, d_U .

Example 5.3 (Using the Durbin–Watson table)

A regression with an intercept and one explanatory variable ($k' = 1$) is fitted to $n = 20$ observations. From the Durbin–Watson table at the 5% level, $d_L = 1.201$ and $d_U = 1.411$.

- If $d = 1.10$: since $1.10 < 1.201 = d_L$, reject H_0 . There is positive autocorrelation.
- If $d = 1.30$: since $1.201 \leq 1.30 \leq 1.411$, the test is inconclusive.
- If $d = 1.90$: since $1.411 < 1.90 < 4 - 1.411 = 2.589$, do not reject H_0 .
- If $d = 2.85$: here $4 - d = 1.15 < 1.201$, so reject H_0 in favour of negative autocorrelation.

With two explanatory variables ($k' = 2$) and $n = 20$, the table gives $d_L = 1.100$ and $d_U = 1.537$. The inconclusive region becomes wider when more regressors are used.

Limitations.

- (i) There are inconclusive regions.
- (ii) The test is for AR(1) only.
- (iii) It is not valid when y_{t-1} is a regressor; then d is biased towards 2 and autocorrelation may be missed (use Durbin's h test below).
- (iv) It requires an intercept.

Exam tip

Draw the d -scale with its five regions: 0, d_L , d_U , 2 , $4 - d_U$, $4 - d_L$, 4 . Examiners expect this diagram, and it makes the decision rule clear.

Practice

- Exercises 5.4 and 5.5 (Exercises and Solutions booklet).

Next lecture

Lecture 36: Durbin's h test.