

Lecture 36: Durbin's h test

Econometrics ■ Unit III: Autocorrelation and instrumental variables

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Previous lecture

Lecture 35: The Durbin–Watson test: decision rule, examples and limitations.

Today

- Detection of autocorrelation
 - Durbin's h test (when y_{t-1} is a regressor)

Durbin's h test (when y_{t-1} is a regressor)

Models with a lagged dependent variable are common in economics, for example

$$y_t = \alpha + \gamma y_{t-1} + \beta x_t + u_t, \quad u_t = \rho u_{t-1} + v_t.$$

Here the Durbin–Watson test cannot be used, because d tends to be close to 2 even when $\rho \neq 0$. Durbin (1970) suggested the following large-sample test of $H_0 : \rho = 0$.

Procedure.

1. Estimate the model by OLS. Note the estimated variance of the coefficient of y_{t-1} , written $\widehat{\text{Var}}(\hat{\gamma})$ (the square of its standard error).
2. Estimate ρ from the residuals, for example by $\hat{\rho} = 1 - d/2$.

3. Compute

$$h = \hat{\rho} \sqrt{\frac{n}{1 - n \widehat{\text{Var}}(\hat{\gamma})}}.$$

4. For large n , h is approximately $N(0, 1)$ under H_0 . For a test against positive autocorrelation at the 5% level, reject H_0 if $h > 1.645$. For a two-sided test, reject if $|h| > 1.96$.

Only the variance of the coefficient of y_{t-1} is used, even if the model has other lagged values or other regressors.

When h cannot be computed. If $n \widehat{\text{Var}}(\hat{\gamma}) \geq 1$, the quantity under the square root is not positive. Then use the following regression test.

- Regress the OLS residual e_t on e_{t-1} and on all the regressors of the original model, including y_{t-1} .
- Test whether the coefficient of e_{t-1} is zero by the usual t test. A significant coefficient indicates autocorrelation.

Example 5.4 (Durbin's h)

For $n = 50$ annual observations, the model $y_t = \alpha + \gamma y_{t-1} + \beta x_t + u_t$ gives $\hat{\gamma} = 0.62$ with standard error 0.06, and $d = 1.70$.

Step 1. $\widehat{\text{Var}}(\hat{\gamma}) = 0.06^2 = 0.0036$, so $n\widehat{\text{Var}}(\hat{\gamma}) = 50(0.0036) = 0.18 < 1$.
The test can be used.

Step 2. $\hat{\rho} = 1 - 1.70/2 = 0.15$.

Step 3.

$$h = 0.15 \sqrt{\frac{50}{1 - 0.18}} = 0.15 \sqrt{60.976} = 0.15(7.809) = 1.171.$$

Step 4. Since $1.171 < 1.645$, H_0 is not rejected at the 5% level. There is no significant evidence of positive autocorrelation.

Practice

- Exercise 5.6 (Exercises and Solutions booklet).
- R practical P09: Diagnosing autocorrelation in a Fisher-type regression.

Next lecture

Lecture 37: ρ known: the generalized difference equation and the Prais–Winsten transformation.