

Lecture 39: Durbin's two-step method; the Hildreth–Lu method

Econometrics ■ Unit III: Autocorrelation and instrumental variables

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B.Sc. (Hons) Statistics, Semester V
Academic Session 2026–2027

Previous lecture

Lecture 38: ρ unknown: estimating ρ ; the Cochrane–Orcutt procedure.

Today

- Solution of autocorrelation
 - Durbin's two-step method
 - The Hildreth–Lu (grid search) method

This method, due to Durbin (1960), estimates ρ directly as a regression coefficient.

Step 1 (rearrange the generalized difference equation). From the generalized difference equation (Section 5.5.1 of the notes),

$$y_t - \rho y_{t-1} = \beta_0(1 - \rho) + \beta_1(x_t - \rho x_{t-1}) + v_t.$$

Moving ρy_{t-1} to the right-hand side,

$$y_t = \beta_0(1 - \rho) + \rho y_{t-1} + \beta_1 x_t - \rho \beta_1 x_{t-1} + v_t.$$

Write this as

$$y_t = a_0 + a_1 y_{t-1} + a_2 x_t + a_3 x_{t-1} + v_t,$$

where $a_0 = \beta_0(1 - \rho)$, $a_1 = \rho$, $a_2 = \beta_1$ and $a_3 = -\rho\beta_1$.

Step 2 (first stage). Regress y_t on a constant, y_{t-1} , x_t and x_{t-1} ($t = 2, \dots, n$) by OLS. The estimated coefficient of y_{t-1} is the estimate $\hat{\rho}$.

Step 3 (second stage). Use this $\hat{\rho}$ to form $y_t^* = y_t - \hat{\rho}y_{t-1}$ and $x_t^* = x_t - \hat{\rho}x_{t-1}$, and regress y_t^* on a constant and x_t^* by OLS. Then $\hat{\beta}_0 = \hat{\beta}_0^*/(1 - \hat{\rho})$.

The method is simple, but the first-stage regression has a lagged y among the regressors, so $\hat{\rho}$ is reliable only in large samples. With several explanatory variables, the first stage contains each x and its lag.

The Hildreth–Lu (grid search) method

In the method of Hildreth and Lu (1960), instead of estimating ρ from residuals, we try many values of ρ and keep the best one.

1. Choose a grid of values in $(-1, 1)$, for example $\rho = 0, 0.1, 0.2, \dots, 0.9$ (only positive values if positive autocorrelation is expected).
2. For each value, form $y_t^* = y_t - \rho y_{t-1}$, $x_t^* = x_t - \rho x_{t-1}$, regress y_t^* on a constant and x_t^* , and note the residual sum of squares (RSS).
3. Select the value of ρ with the smallest RSS.
4. Refine: repeat with a finer grid around the selected value (for example, steps of 0.01 or 0.02).
5. Use the final ρ and its regression as the estimates.

Merits

It finds the value of ρ that gives the smallest RSS over the whole range, so it does not get stuck at a poor value. Cochrane–Orcutt, in contrast, may stop at a value that is not the overall best.

Demerit

It needs many regressions, but with a computer this is easy.

Practice

- Exercise 5.8 (Exercises and Solutions booklet).

Next lecture

Lecture 40: A complete worked example; summary of key formulas.