

Lecture 42: Instruments; the IV estimator in the simple regression

Econometrics ■ Unit III: Autocorrelation and instrumental variables

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Previous lecture

Lecture 41: Why OLS fails: consistency; a regressor correlated with the disturbance.

Today

- Instruments
- The IV estimator in the simple regression
 - Derivation from covariances
 - Derivation from the normal (moment) equations

Definition 6.2

A variable z is an **instrumental variable** (instrument) for the regressor x if

1. **Exogeneity:** z is uncorrelated with the disturbance, $\text{Cov}(z, \varepsilon) = 0$; and
2. **Relevance:** z is correlated with x , $\text{Cov}(z, x) \neq 0$.

Both conditions are needed.

- A variable that is uncorrelated with ε but unrelated to x carries no information about β_1 .
- A variable correlated with x but also with ε is not valid.

Example

In a demand equation, price is endogenous. A cost variable of producers, for example the price of raw material, can be an instrument for price.

- It changes supply and hence the market price (relevance).
- But it does not directly change consumers' demand (exogeneity).

Take $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ with instrument z_i .

Step 1. Take the covariance of both sides with z :

$$\text{Cov}(z, y) = \text{Cov}(z, \beta_0) + \beta_1 \text{Cov}(z, x) + \text{Cov}(z, \varepsilon) = 0 + \beta_1 \text{Cov}(z, x) + 0.$$

Step 2. Since $\text{Cov}(z, x) \neq 0$, $\beta_1 = \frac{\text{Cov}(z, y)}{\text{Cov}(z, x)}$.

Step 3. Replace the population covariances by sample covariances:

$$\hat{\beta}_{1,IV} = \frac{\sum_i (z_i - \bar{z})(y_i - \bar{y})}{\sum_i (z_i - \bar{z})(x_i - \bar{x})}, \quad \hat{\beta}_{0,IV} = \bar{y} - \hat{\beta}_{1,IV} \bar{x}.$$

If $z = x$, this becomes the OLS estimator.

Derivation from the normal (moment) equations

OLS solves

$$\sum_i (y_i - b_0 - b_1 x_i) = 0 \quad \text{and} \quad \sum_i x_i (y_i - b_0 - b_1 x_i) = 0.$$

IV replaces the multiplier x_i in the second equation by z_i :

$$\sum_i (y_i - b_0 - b_1 x_i) = 0, \quad \sum_i z_i (y_i - b_0 - b_1 x_i) = 0.$$

The first gives $b_0 = \bar{y} - b_1 \bar{x}$. Substituting into the second,

$$\sum_i z_i \{(y_i - \bar{y}) - b_1 (x_i - \bar{x})\} = 0 \implies b_1 = \frac{\sum_i z_i (y_i - \bar{y})}{\sum_i z_i (x_i - \bar{x})},$$

which equals $\hat{\beta}_{1,IV}$ because $\sum_i \bar{z} (y_i - \bar{y}) = 0$ and $\sum_i \bar{z} (x_i - \bar{x}) = 0$.

Practice

- Exercises 6.2 and 6.4 (Exercises and Solutions booklet).

Next lecture

Lecture 43: Consistency of the IV estimator; an IV calculation.