# Lecture 43: Consistency of the IV estimator; an IV calculation

Econometrics ▪ Unit III: Autocorrelation and instrumental variables

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B.Sc. (Hons) Statistics, Semester V
Academic Session 2026–2027

# This lecture

## Previous lecture

Lecture 42: Instruments; the IV estimator in the simple regression.

## Today

- The IV estimator in the simple regression
  - Consistency

# Consistency of the IV estimator

Substituting $y_i - \bar{y} = \beta_1(x_i - \bar{x}) + (\varepsilon_i - \bar{\varepsilon})$,

$$\widehat{\beta}_{1,IV} = \beta_1 + \frac{\frac{1}{n}\sum_i (z_i - \bar{z})\varepsilon_i}{\frac{1}{n}\sum_i (z_i - \bar{z})(x_i - \bar{x})}.$$

Taking probability limits,

$$\operatorname{plim} \widehat{\beta}_{1,IV} = \beta_1 + \frac{\operatorname{Cov}(z, \varepsilon)}{\operatorname{Cov}(z, x)} = \beta_1 + \frac{0}{\operatorname{Cov}(z, x)} = \beta_1.$$

- So the IV estimator is consistent.
- It is generally *not* unbiased in finite samples.

# An IV calculation

## Example 6.2 (An IV calculation)

| $i$ | $z_i$ | $x_i$ | $y_i$ |
|---|---|---|---|
| 1 | $-2$ | 1 | 1 |
| 2 | $-1$ | 2 | 3 |
| 3 | 0 | 2 | 2 |
| 4 | 1 | 4 | 5 |
| 5 | 2 | 6 | 9 |

**Means.** $\bar{z} = 0$, $\bar{x} = 15/5 = 3$, $\bar{y} = 20/5 = 4$.

**Cross products.** Since $\bar{z} = 0$,

$$\begin{aligned} \sum z_i(x_i - \bar{x}) &= \sum z_i x_i \\ &= -2 - 2 + 0 + 4 + 12 = 12, \\ \sum z_i(y_i - \bar{y}) &= \sum z_i y_i \\ &= -2 - 3 + 0 + 5 + 18 = 18. \end{aligned}$$

**Estimates.** $\widehat{\beta}_{1,IV} = 18/12 = 1.5$ and $\widehat{\beta}_{0,IV} = 4 - 1.5(3) = -0.5$.

## Example 6.2 (cont.)

**Comparison with OLS.**

$$\sum (x_i - \bar{x})^2 = 4 + 1 + 1 + 1 + 9 = 16,$$
$$\sum (x_i - \bar{x})(y_i - \bar{y}) = 6 + 1 + 2 + 1 + 15 = 25,$$

so $b_1 = 25/16 = 1.5625$ and $b_0 = 4 - 1.5625(3) = -0.6875$.

**Residual check.** The IV residuals $y_i + 0.5 - 1.5x_i$ are

$$(0,\ 0.5,\ -0.5,\ -0.5,\ 0.5).$$

They sum to zero and $\sum z_i e_i = 0 - 0.5 + 0 - 0.5 + 1 = 0$, as required by the IV equations.

# After this lecture

## Practice

- Exercise 6.3 (Exercises and Solutions booklet).

## Next lecture

Lecture 44: The IV estimator in matrix form: consistency and variance.