

Lecture 44: The IV estimator in matrix form: consistency and variance

Econometrics ■ Unit III: Autocorrelation and instrumental variables

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Previous lecture

Lecture 43: Consistency of the IV estimator; an IV calculation.

Today

- The IV estimator in matrix form

The IV estimator in matrix form

Let X be $n \times k$ (including the constant) and let Z be an $n \times k$ matrix of instruments.

- Regressors that are uncorrelated with ε (including the constant) serve as their own instruments;
- each endogenous regressor is replaced by an instrument.

Step 1 (moment equations). The IV estimator solves $Z'(y - Xb) = 0$, the sample analogue of $E(Z'\varepsilon) = 0$.

Step 2 (solve). $Z'y - Z'Xb = 0 \Rightarrow Z'Xb = Z'y$. If $Z'X$ is nonsingular,

$$b_{IV} = (Z'X)^{-1}Z'y.$$

Step 3 (consistency). Substituting $y = X\beta + \epsilon$,

$$b_{IV} = (Z'X)^{-1}Z'X\beta + (Z'X)^{-1}Z'\epsilon = \beta + \left(\frac{Z'X}{n}\right)^{-1} \frac{Z'\epsilon}{n}.$$

Assume $\text{plim } Z'X/n = Q_{ZX}$ is finite and nonsingular (relevance) and $\text{plim } Z'\epsilon/n = 0$ (exogeneity). Then

$$\text{plim } b_{IV} = \beta + Q_{ZX}^{-1} \cdot 0 = \beta.$$

Step 4 (variance). If the disturbances have constant variance σ^2 and are uncorrelated, the approximate (large-sample) covariance matrix is

$$\text{Var}(b_{IV}) \approx \sigma^2 (Z'X)^{-1} Z'Z (X'Z)^{-1}.$$

The covariance matrix is estimated by replacing σ^2 with

$$\hat{\sigma}^2 = \frac{(y - Xb_{IV})'(y - Xb_{IV})}{n - k}.$$

The residuals must be computed with the original X and the IV estimates.

Remember

$$b_{IV} = (Z'X)^{-1}Z'y; \quad \text{simple case } \hat{\beta}_{1,IV} = \frac{\sum(z_i - \bar{z})(y_i - \bar{y})}{\sum(z_i - \bar{z})(x_i - \bar{x})}.$$

Practice

Re-work the derivations and examples of this lecture without looking at the slides.

Next lecture

Lecture 45: Relevance and weak instruments; more instruments than regressors; summary.