

Lecture 45: Relevance and weak instruments; more instruments than regressors; summary

Econometrics ■ Unit III: Autocorrelation and instrumental variables

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Previous lecture

Lecture 44: The IV estimator in matrix form: consistency and variance.

Today

- The IV estimator in matrix form
 - Relevance and precision
- More instruments than regressors
- Summary of key formulas

In the simple regression with an exogenous x , the OLS slope has variance about $\sigma^2/\{n \text{Var}(x)\}$. The IV slope has variance about

$$\frac{\sigma^2 \text{Var}(z)}{n\{\text{Cov}(z, x)\}^2} = \frac{\sigma^2}{n \text{Var}(x) r_{zx}^2},$$

where r_{zx} is the correlation between z and x ; the second form follows from

$$\{\text{Cov}(z, x)\}^2 = r_{zx}^2 \text{Var}(z) \text{Var}(x).$$

Since $r_{zx}^2 \leq 1$, IV is less precise than OLS would be if x were exogenous.

- If r_{zx} is small (a **weak instrument**), the variance is very large.
- For example, reducing $|r_{zx}|$ from 0.8 to 0.2 multiplies the variance by $0.64/0.04 = 16$.
- IV is used for its consistency, not for efficiency.

Exam tip

Write both conditions as formulas, $\text{Cov}(z, \varepsilon) = 0$ (exogeneity) and $\text{Cov}(z, x) \neq 0$ (relevance), and give an economic example of an instrument.

If there are more instruments than coefficients, the equations

$$Z'(y - Xb) = 0$$

cannot all be satisfied exactly.

- The instruments are then combined by two-stage least squares, which is developed in Chapter 8.

- If $\text{Cov}(x, \varepsilon) \neq 0$: $\text{plim } b_1 = \beta_1 + \text{Cov}(x, \varepsilon) / \text{Var}(x)$ (OLS inconsistent).
- Instrument z : $\text{Cov}(z, \varepsilon) = 0$ (exogeneity) and $\text{Cov}(z, x) \neq 0$ (relevance).
- Simple IV: $\hat{\beta}_{1,IV} = \sum(z_i - \bar{z})(y_i - \bar{y}) / \sum(z_i - \bar{z})(x_i - \bar{x})$,
 $\hat{\beta}_{0,IV} = \bar{y} - \hat{\beta}_{1,IV}\bar{x}$.
- Matrix IV: $b_{IV} = (Z'X)^{-1}Z'y$, consistent;
 $\text{Var}(b_{IV}) \approx \sigma^2(Z'X)^{-1}Z'Z(X'Z)^{-1}$.
- Precision: $\text{Var}(\hat{\beta}_{1,IV}) \approx \sigma^2 / \{n \text{Var}(x) r_{zx}^2\}$; weak instruments give large variance.

Practice

- Exercise 6.5 (Exercises and Solutions booklet).
- R practical P11: Instrument relevance and endogeneity in the world copper market.

Next lecture

Lecture 46 begins Unit IV: Simultaneous equations models.