Lecture 47: A demand–supply system: structural and reduced forms

Econometrics ▪ Unit IV: Simultaneous equations models

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This lecture

Previous lecture

Lecture 46: Simultaneous equations models; endogenous, exogenous and predetermined variables.

Today

- A demand–supply system: structural and reduced forms

A demand–supply system

Consider

$$\text{Demand:} \quad Q_t = \alpha_0 + \alpha_1 P_t + \alpha_2 I_t + u_{Dt}, \tag{7.1}$$

$$\text{Supply:} \quad Q_t = \beta_0 + \beta_1 P_t + \beta_2 W_t + u_{St}, \tag{7.2}$$

where Q_t is quantity, P_t price, I_t consumers' income and W_t an input cost (for example, the wage rate) of producers.

- Q_t and P_t are endogenous; I_t and W_t are exogenous.
- Theory suggests $\alpha_1 < 0$, $\alpha_2 > 0$, $\beta_1 > 0$ and $\beta_2 < 0$.
- Income is excluded from supply and input cost is excluded from demand.

Deriving the reduced form

Step 1 (equilibrium). Equate demand and supply:

$$\alpha_0 + \alpha_1 P_t + \alpha_2 I_t + u_{Dt} = \beta_0 + \beta_1 P_t + \beta_2 W_t + u_{St}.$$

Step 2 (solve for price). Collect the P_t terms on one side:

$$(\beta_1 - \alpha_1) P_t = (\alpha_0 - \beta_0) + \alpha_2 I_t - \beta_2 W_t + (u_{Dt} - u_{St}).$$

With $\Delta = \beta_1 - \alpha_1 \neq 0$,

$$P_t = \frac{\alpha_0 - \beta_0}{\Delta} + \frac{\alpha_2}{\Delta} I_t - \frac{\beta_2}{\Delta} W_t + \frac{u_{Dt} - u_{St}}{\Delta}.$$

Deriving the reduced form (cont.)

Step 3 (solve for quantity). Substitute into demand:

$$
\begin{aligned}
Q_t &= \alpha_0 + \alpha_1\left\{\frac{\alpha_0 - \beta_0}{\Delta} + \frac{\alpha_2}{\Delta} I_t - \frac{\beta_2}{\Delta} W_t + \frac{u_{Dt} - u_{St}}{\Delta}\right\} + \alpha_2 I_t + u_{Dt} \\
&= \frac{\alpha_0\Delta + \alpha_1\alpha_0 - \alpha_1\beta_0}{\Delta} + \frac{\alpha_1\alpha_2 + \alpha_2\Delta}{\Delta} I_t \\
&\quad - \frac{\alpha_1\beta_2}{\Delta} W_t + \frac{\alpha_1 u_{Dt} - \alpha_1 u_{St} + \Delta u_{Dt}}{\Delta}.
\end{aligned}
$$

Since $\Delta + \alpha_1 = \beta_1$,

$$
Q_t = \frac{\beta_1\alpha_0 - \alpha_1\beta_0}{\Delta} + \frac{\beta_1\alpha_2}{\Delta} I_t - \frac{\alpha_1\beta_2}{\Delta} W_t + \frac{\beta_1 u_{Dt} - \alpha_1 u_{St}}{\Delta}.
$$

The reduced form

These two equations express each endogenous variable in terms of exogenous variables and disturbances only. They form the **reduced form**:

$$P_t = \pi_{10} + \pi_{11} I_t + \pi_{12} W_t + v_{1t},$$
$$Q_t = \pi_{20} + \pi_{21} I_t + \pi_{22} W_t + v_{2t}.$$

- The coefficients π_{ij} are **reduced-form coefficients**; each is a combination of structural coefficients.
- Since the right side contains only exogenous variables, each reduced-form equation can be estimated consistently by OLS.

Price is correlated with the demand disturbance. The reduced form shows that P_t contains $(u_{Dt} - u_{St})/\Delta$. So P_t is correlated with u_{Dt} (and u_{St}). This is why OLS applied to a structural equation is inconsistent (Chapter 8).

Example 7.1 (Solving a market model)

Let

$$\text{Demand: } Q_t = 20 - 2P_t + 0.5I_t + u_{Dt},$$
$$\text{Supply: } Q_t = 2 + P_t - W_t + u_{St}.$$

Here $\alpha_1 = -2$, $\beta_1 = 1$, so $\Delta = 1 - (-2) = 3$. Using the formulas:

$$P_t = \frac{20-2}{3} + \frac{0.5}{3} I_t - \frac{(-1)}{3} W_t + \frac{u_{Dt} - u_{St}}{3}$$
$$= 6 + \frac{1}{6} I_t + \frac{1}{3} W_t + \frac{u_{Dt} - u_{St}}{3}.$$

A market model (cont.)

Example 7.1 (cont.)

$$Q_t = \frac{1(20) - (-2)(2)}{3} + \frac{1(0.5)}{3} I_t - \frac{(-2)(-1)}{3} W_t + \frac{u_{Dt} + 2u_{St}}{3}$$
$$= 8 + \frac{1}{6} I_t - \frac{2}{3} W_t + \frac{u_{Dt} + 2u_{St}}{3}.$$

Check. For $I_t = 12$, $W_t = 3$ and zero disturbances:

$$P_t = 6 + 2 + 1 = 9 \quad \text{and} \quad Q_t = 8 + 2 - 2 = 8.$$

Demand gives $20 - 18 + 6 = 8$ and supply gives $2 + 9 - 3 = 8$. Both agree.

After this lecture

Practice

- Exercise 7.2 (Exercises and Solutions booklet).

Next lecture

Lecture 48: General form of the system; the problem of identification.