

Lecture 48: General form of the system; the problem of identification

Econometrics ■ Unit IV: Simultaneous equations models

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Previous lecture

Lecture 47: A demand–supply system: structural and reduced forms.

Today

- General form of the system
- The problem of identification
 - Why identification can fail
 - How exclusions identify an equation

With G endogenous variables $\mathbf{y}_t$ and K exogenous variables $\mathbf{x}_t$ (including the constant), write all variables on the left:

$$B\mathbf{y}_t + \Gamma\mathbf{x}_t = \mathbf{u}_t,$$

where B is $G \times G$ and nonsingular and Γ is $G \times K$. Each row is one structural equation.

Premultiplying by B^{-1} :

$$\mathbf{y}_t + B^{-1}\Gamma\mathbf{x}_t = B^{-1}\mathbf{u}_t \implies \mathbf{y}_t = \Pi\mathbf{x}_t + \mathbf{v}_t,$$

$$\Pi = -B^{-1}\Gamma, \quad \mathbf{v}_t = B^{-1}\mathbf{u}_t.$$

This is the reduced form in matrix notation.

The market model in matrix form

For the market model, with $\mathbf{y}_t = (Q_t, P_t)'$ and $\mathbf{x}_t = (1, I_t, W_t)'$, the equations written with all variables on the left are

$$Q_t - \alpha_1 P_t - \alpha_0 - \alpha_2 I_t = u_{Dt}, \quad Q_t - \beta_1 P_t - \beta_0 - \beta_2 W_t = u_{St},$$

so the full coefficient matrix $[B \ \Gamma]$, with columns $(Q, P, 1, I, W)$, is

$$[B \ \Gamma] = \begin{pmatrix} 1 & -\alpha_1 & -\alpha_0 & -\alpha_2 & 0 \\ 1 & -\beta_1 & -\beta_0 & 0 & -\beta_2 \end{pmatrix}.$$

- The zeros record the excluded variables.
- Setting the coefficient of the left-hand endogenous variable equal to one is called **normalisation**.

Definition 7.1

A structural equation is **identified** if its parameters can be uniquely determined from the reduced-form parameters. It is

- **unidentified (under-identified)** if they cannot be determined;
 - **exactly identified** if they can be determined in exactly one way;
 - **over-identified** if they can be determined in more than one way (that is, there is more information than needed).
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- The reduced form can always be estimated from data.
 - The question is whether the structural parameters can be recovered from it. If not, no amount of data will give them.

Why identification can fail

Suppose demand and supply contain only price:

$$\text{Demand: } Q = a_0 + a_1P + u_D, \quad \text{Supply: } Q = b_0 + b_1P + u_S.$$

Multiply demand by any constant c and supply by $1 - c$, and add:

$$Q = \{ca_0 + (1 - c)b_0\} + \{ca_1 + (1 - c)b_1\}P + \{cu_D + (1 - c)u_S\}.$$

- This “mixed” equation has exactly the same variables as the demand and supply equations.
- When we fit a line to observed (P, Q) points, we cannot tell whether we have estimated demand, supply or some mixture.
- Neither equation is identified.

How exclusions identify an equation

In the market example, hold $I = 12$ and the disturbances at zero.

When the input cost rises from $W = 3$ to $W = 6$, the reduced form gives:

$$P = 6 + 2 + 2 = 10, \quad Q = 8 + 2 - 4 = 6.$$

- The equilibrium moves from $(9, 8)$ to $(10, 6)$, a slope of

$$\frac{6 - 8}{10 - 9} = -2,$$

which is the demand slope α_1 .

- Input cost shifts the supply curve, and because W is excluded from demand, the moving equilibrium traces out the demand curve.

How exclusions identify an equation (cont.)

Similarly, raising income from 12 to 18 (with $W = 3$) moves the equilibrium to

$$P = 6 + 3 + 1 = 10, \quad Q = 8 + 3 - 2 = 9,$$

a slope of $1 = \beta_1$: income shifts demand and traces out supply.

So a variable excluded from one equation but present in another helps to identify the first equation.

Practice

- Exercise 7.3 (Exercises and Solutions booklet).

Next lecture

Lecture 49: The order condition.