Lecture 50: The rank condition; working procedure for identification

Econometrics ▪ Unit IV: Simultaneous equations models

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This lecture

Previous lecture

Lecture 49: The order condition.

Today

- The rank condition (necessary and sufficient condition)
- Working procedure for identification

Constructing the matrix A_i

Construction. Write the full coefficient matrix $[B\ \Gamma]$ with one row per equation and one column per variable. For equation i:

1. Select the columns of the variables *excluded* from equation i (those with a zero in row i).
2. Delete row i (which is all zeros in these columns).
3. Call the remaining matrix A_i. It has $G - 1$ rows.

The rank condition

Theorem 7.3 (Rank condition)

Equation i is identified ***if and only if***

$$\operatorname{rank}(A_i) = G - 1,$$

that is, at least one $(G-1) \times (G-1)$ determinant formed from A_i is nonzero. Hence the rank condition is both ***necessary and sufficient*** *for identification.*

Idea of the proof.

Any linear combination of the G equations is consistent with the same data (as in the "mixed" equation above). Equation i is identified if the only combination that has the *same zeros* as equation i is equation i itself (up to normalisation).

Idea of the proof (cont.)

Step 1. Let $a = (a_1, \ldots, a_G)'$ be the weights of a combination. In the excluded columns, the combination has entries $a'H_S$, where H_S is the matrix of selected columns (before deleting row i). It has the same zeros as equation i exactly when $a'H_S = 0$.

Step 2. Row i of H_S is zero, so $a'H_S = 0$ involves only the other weights:

$$\sum_{l \neq i} a_l \times (\text{row } l \text{ of } H_S) = 0, \quad \text{that is,} \quad (a_l)'_{l \neq i} A_i = 0.$$

The rank condition: idea of the proof (cont.)

Idea of the proof (cont.)

Step 3. If $\operatorname{rank}(A_i) = G - 1$, the $G - 1$ rows of A_i are linearly independent, so all $a_l = 0$ for $l \neq i$. The only admissible combination is a multiple of equation i; after normalisation it is equation i itself. So the equation is identified.

Step 4. If $\operatorname{rank}(A_i) < G - 1$, the rows are dependent and some nonzero $(a_l)_{l \neq i}$ satisfies Step 2. Adding this combination of other equations to equation i gives a different equation with the same zeros and the same reduced form. So the equation is not identified. □

Working procedure for identification

1. Write every equation with all variables on the left and normalise.
2. Count G and K for the system and g, k for the equation.
3. Check the order condition. If it fails, stop: the equation is unidentified.
4. Form A_i and check whether $\mathrm{rank}(A_i) = G - 1$.
5. If the rank condition holds: the equation is exactly identified when $K - k = g - 1$ and over-identified when $K - k > g - 1$. If it fails, the equation is unidentified.

The entries of A_i must be actual (nonzero) coefficients. A variable written in another equation does not help if its true coefficient is zero.

After this lecture

Practice

- Exercise 7.5 (Exercises and Solutions booklet).

Next lecture

Lecture 51: The X/0 table method.