

Lecture 51: The X/0 table method

Econometrics ■ Unit IV: Simultaneous equations models

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Previous lecture

Lecture 50: The rank condition; working procedure for identification.

Today

- A quick table method (the X/0 table)

The X/0 table

For larger systems, the rank condition can be checked quickly with a table of symbols, without writing the coefficients.

Step 1 (make the table). Write one row for each equation and one column for each variable of the system (endogenous first, then exogenous). Put X if the variable appears in the equation and 0 if it does not. A constant term that appears in every equation can be left out, because it is never excluded and it cancels in $K - k$.

Step 2 (order condition). For the equation under study, count g (X's among the endogenous columns) and k (X's among the exogenous columns), and compute the **degree of over-identification**

$$L = (K - k) - (g - 1).$$

If $L < 0$, the equation is unidentified; stop.

Step 3 (the small table). Keep only the columns in which the equation under study has 0, and delete its own row. This leaves a table with $G - 1$ rows.

Step 4 (rank condition).

- If some row of the small table is entirely 0, the rank is less than $G - 1$: the equation is **not identified**. (That other equation contains none of the excluded variables, so nothing separates it from the equation under study.)
- Otherwise, try to choose $G - 1$ columns of the small table so that you can pick one X in each row, each from a different column. If this is possible, the rank is $G - 1$ (unless the actual coefficients happen to make the determinant zero), and the equation is **identified**.

Step 5 (status). If identified: $L = 0$ means exactly identified and $L > 0$ means over-identified.

The X/0 table works with symbols only. When numerical coefficients are known, confirm Step 4 by computing a determinant, because special values (as in the second example of the Examples section, Example 7.4) can make it zero.

Example 7.2 (Using the X/0 table)

A system has three endogenous variables y_1, y_2, y_3 and three exogenous variables x_1, x_2, x_3 (plus a constant in every equation). So $G = 3$ and $K = 3$.

Equation	y_1	y_2	y_3	x_1	x_2	x_3	g	k	$L = (K - k) - (g - 1)$
1	X	X	0	X	0	0	2	1	$2 - 1 = 1$
2	X	X	X	X	0	0	3	1	$2 - 2 = 0$
3	0	X	X	0	X	X	2	2	$1 - 1 = 0$

Example 7.2 (cont.)

Equation 1. $L = 1 \geq 0$. Its zeros are in columns y_3, x_2, x_3 . The small table is

	y_3	x_2	x_3
Eq. 2	X	0	0
Eq. 3	X	X	X

Choose columns y_3 and x_2 : the row of Eq. 2 has X under y_3 , and the row of Eq. 3 has X under x_2 . So the rank is $2 = G - 1$. Equation 1 is **over-identified** ($L = 1$).

Example 7.2 (cont.)

Equation 2. $L = 0$, so the order condition holds. Its zeros are in columns x_2, x_3 :

	x_2	x_3
Eq. 1	0	0
Eq. 3	X	X

The row of equation 1 is entirely 0, so the rank is at most 1, which is less than 2. Equation 2 is **not identified**, although the order condition holds. Indeed, equation 1 contains neither x_2 nor x_3 , so any combination of equations 1 and 2 has the same zeros as equation 2.

Example 7.2 (cont.)

Equation 3. $L = 0$. Its zeros are in columns y_1, x_1 :

	y_1	x_1
Eq. 1	X	X
Eq. 2	X	X

We can pick X under y_1 in the row of Eq. 1 and X under x_1 in the row of Eq. 2. The rank is 2, unless the coefficients of y_1 and x_1 happen to be in the same ratio in both equations. Equation 3 is **exactly identified**.

Exam tip

When asked for the “necessary and sufficient condition for identifiability”, state both conditions: the order condition is necessary only, and the rank condition is necessary and sufficient. Always check the rank condition after the order condition.

Practice

- Exercise 7.6 (Exercises and Solutions booklet).

Next lecture

Lecture 52: Identification step by step: the market for milk.