

Lecture 52: Identification step by step: the market for milk

Econometrics ■ Unit IV: Simultaneous equations models

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This lecture

Previous lecture

Lecture 51: The X/0 table method.

Today

- Building up identification step by step: the market for milk

The market for milk: Stage A

This example shows how adding exogenous variables to the right equations makes a model identified. Let Q_t be the quantity of milk sold in a town and P_t its price; both are endogenous.

Stage A: price only.

$$\text{Demand: } Q_t = a_0 + a_1 P_t + u_{1t}, \quad \text{Supply: } Q_t = b_0 + b_1 P_t + u_{2t}.$$

- Here $G = 2$ and $K = 1$ (the constant). For each equation $g = 2$, $k = 1$, so $L = (1 - 1) - (2 - 1) = -1 < 0$.
- *Neither equation is identified.* The observed points are only the equilibrium points, and any mixture of the two equations fits them equally well.

Stage B: add consumers' income I_t to demand. Income affects how much milk families buy, but not how much the dairy farmers produce.

$$\text{Demand: } Q_t = a_0 + a_1 P_t + a_2 I_t + u_{1t},$$

$$\text{Supply: } Q_t = b_0 + b_1 P_t + u_{2t}.$$

Now $K = 2$.

- Demand: $k = 2$, so $L = (2 - 2) - 1 = -1$: *not identified*.
- Supply: $k = 1$, so $L = (2 - 1) - 1 = 0$, and the excluded variable I appears in demand ($a_2 \neq 0$): *exactly identified*.

When income changes, the demand curve shifts while the supply curve stays fixed, so the equilibrium points move along the supply curve and trace it out.

Stage C: add the price of cattle feed F_t to supply. Costlier feed reduces the milk farmers are willing to supply, but does not change consumers' demand.

$$\text{Demand: } Q_t = a_0 + a_1 P_t + a_2 I_t + u_{1t},$$

$$\text{Supply: } Q_t = b_0 + b_1 P_t + b_2 F_t + u_{2t}.$$

- Now $K = 3$ and, for each equation, $k = 2$ and $L = (3 - 2) - 1 = 0$.
- Demand excludes F (present in supply) and supply excludes I (present in demand). *Both equations are exactly identified.*
- Changes in feed price shift supply and trace out the demand curve.

Stage D: add the number of milch animals A_t to supply as well.

$$\text{Demand: } Q_t = a_0 + a_1 P_t + a_2 I_t + u_{1t},$$

$$\text{Supply: } Q_t = b_0 + b_1 P_t + b_2 F_t + b_3 A_t + u_{2t}.$$

Now $K = 4$.

- Demand: $k = 2$, $L = (4 - 2) - 1 = 1$. It excludes F and A , and at least one of them is in supply: *over-identified*. Two different supply shifters each trace out the demand curve, so the demand slope can be found in two ways from the reduced form.
- Supply: $k = 3$, $L = (4 - 3) - 1 = 0$, and I is in demand: *exactly identified*.

The market for milk: summary

Stage	Exogenous variables	Demand	Supply
A	constant	not identified	not identified
B	constant, I	not identified	exactly identified
C	constant, I , F	exactly identified	exactly identified
D	constant, I , F , A	over-identified	exactly identified

Lesson.

- An equation is identified by the variables it *excludes* that appear elsewhere in the system.
- A variable that shifts *one* curve only helps to identify the *other* curve.

Practice

Re-work the derivations and examples of this lecture without looking at the slides.

Next lecture

Lecture 53: Examples of identification; summary of key points.