

Lecture 53: Examples of identification; summary of key points

Econometrics ■ Unit IV: Simultaneous equations models

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This lecture

Previous lecture

Lecture 52: Identification step by step: the market for milk.

Today

- Examples
- Summary of key points

Example 7.3 (The demand–supply system)

From the matrix $[B \ \Gamma]$ with columns $(Q, P, 1, I, W)$ above: $G = 2$, $K = 3$ (constant, I , W).

Demand. $g = 2$ (Q, P), $k = 2$ (constant, I). Order: $K - k = 1 = g - 1$; may be exactly identified. Excluded variable: W . Its column, after deleting the demand row, is $A_1 = (-\beta_2)$. Since $G - 1 = 1$, we need $\text{rank}(A_1) = 1$, which holds if $\beta_2 \neq 0$. Demand is **exactly identified**.

Supply. $g = 2$, $k = 2$ (constant, W). Order: $K - k = 1 = g - 1$. Excluded: I , giving $A_2 = (-\alpha_2)$, of rank 1 if $\alpha_2 \neq 0$. Supply is **exactly identified**.

Example 7.3 (cont.)

Recovering the structural parameters. Exclusion of W from demand means that, in the reduced form, the effects of W on Q and on P must be in the ratio α_1 :

$$\alpha_1 = \frac{\pi_{22}}{\pi_{12}} = \frac{-\alpha_1\beta_2/\Delta}{-\beta_2/\Delta}. \checkmark$$

Then $\alpha_2 = \pi_{21} - \alpha_1\pi_{11}$ and $\alpha_0 = \pi_{20} - \alpha_1\pi_{10}$.

Similarly, for supply, $\beta_1 = \pi_{21}/\pi_{11}$, $\beta_2 = \pi_{22} - \beta_1\pi_{12}$ and $\beta_0 = \pi_{20} - \beta_1\pi_{10}$.

In the numerical model, $\alpha_1 = (-2/3)/(1/3) = -2$ and $\beta_1 = (1/6)/(1/6) = 1$.

Example 7.4 (Order condition holds but rank condition fails)

Consider the system (no constant)

$$\begin{aligned}y_1 - y_2 - y_3 &= u_1, \\y_2 - x_1 - 2x_2 &= u_2, \\y_3 - 2x_1 - 4x_2 &= u_3.\end{aligned}$$

Here $G = 3$, $K = 2$. For the first equation, $g = 3$, $k = 0$.

Order. $K - k = 2 = g - 1$: may be exactly identified.

Example 7.4 (cont.)

Rank. Excluded from equation 1: x_1, x_2 . Their coefficients in equations 2 and 3 are

$$A_1 = \begin{pmatrix} -1 & -2 \\ -2 & -4 \end{pmatrix}, \quad |A_1| = 4 - 4 = 0.$$

So $\text{rank}(A_1) = 1 < G - 1 = 2$: the first equation is **unidentified**.

Why. Subtracting twice equation 2 from equation 3 gives

$$y_3 - 2y_2 = u_3 - 2u_2,$$

which contains no exogenous variable. Adding any multiple of it to equation 1 keeps the same exclusions, so the data cannot distinguish the resulting equations.

Example 7.5 (Restoring identification)

Change the third equation to $y_3 - 2x_1 - 5x_2 = u_3$. Now

$$A_1 = \begin{pmatrix} -1 & -2 \\ -2 & -5 \end{pmatrix}, \quad |A_1| = 5 - 4 = 1 \neq 0,$$

so the first equation is exactly identified.

Directly: the reduced forms are

$$y_2 = x_1 + 2x_2 + u_2 \quad \text{and} \quad y_3 = 2x_1 + 5x_2 + u_3,$$

so $y_1 = y_2 + y_3 + u_1$ has reduced-form coefficients $(1 + 2, 2 + 5) = (3, 7)$.

Example 7.5 (cont.)

If the structural slopes of y_2, y_3 are unknown, b_2 and b_3 , then

$$b_2 + 2b_3 = 3, \quad 2b_2 + 5b_3 = 7.$$

Subtracting twice the first from the second gives $b_3 = 1$, and then $b_2 = 1$: a unique solution.

Example 7.6 (An over-identified equation)

Add a second exogenous supply shifter R_t (rainfall) to supply:

$$Q_t = \alpha_0 + \alpha_1 P_t + \alpha_2 I_t + u_{Dt},$$

$$Q_t = \beta_0 + \beta_1 P_t + \beta_2 W_t + \beta_3 R_t + u_{St}.$$

Now $G = 2$, $K = 4$ (constant, I , W , R).

Demand. $g = 2$, $k = 2$, so $K - k = 2 > g - 1 = 1$. Excluded: W, R , giving $A_1 = (-\beta_2, -\beta_3)$, of rank 1 if β_2 or β_3 is nonzero. Demand is **over-identified**.

Example 7.6 (cont.)

Let π_{13} and π_{23} be the reduced-form coefficients of R_t in the price and quantity equations. Then both ratios π_{22}/π_{12} (from W) and π_{23}/π_{13} (from R) equal α_1 . So α_1 can be obtained in two ways.

Supply. $g = 2$, $k = 3$, so $K - k = 1 = g - 1$, and $A_2 = (-\alpha_2)$. Supply is **exactly identified** if $\alpha_2 \neq 0$.

Different equations of one system can have different identification status.

- Structural form $B\mathbf{y}_t + \Gamma\mathbf{x}_t = \mathbf{u}_t$; reduced form $\mathbf{y}_t = \Pi\mathbf{x}_t + \mathbf{v}_t$, $\Pi = -B^{-1}\Gamma$.
- Endogenous variables are correlated with the structural disturbances; reduced forms can be estimated by OLS.
- Identification: structural parameters can be found from reduced-form parameters (unidentified, exactly identified, over-identified).
- Order condition (necessary): $K - k \geq g - 1$; degree of over-identification $L = (K - k) - (g - 1)$.
- Rank condition (necessary and sufficient): $\text{rank}(A_i) = G - 1$, where A_i holds the coefficients, in the other equations, of the variables excluded from equation i .
- X/0 table: a row of zeros in the small table means not identified.

Practice

- R practical P12: Structural form, reduced form and identification.

Next lecture

Lecture 54: OLS in a structural equation: inconsistency and where it is appropriate.