

Lecture 54: OLS in a structural equation: inconsistency and where it is appropriate

Econometrics ■ Unit IV: Simultaneous equations models

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Previous lecture

Lecture 53: Examples of identification; summary of key points.

Today

- Notation for one structural equation
- Ordinary least squares
 - OLS is inconsistent for a structural equation
 - Where OLS is appropriate

We estimate one identified structural equation at a time. Write it as

$$y_1 = Y_1\beta + X_1\gamma + u = D\theta + u, \quad D = [Y_1 \ X_1], \quad \theta = \begin{pmatrix} \beta \\ \gamma \end{pmatrix}, \quad (8.1)$$

where

- y_1 ($n \times 1$) is the left-hand endogenous variable;
- Y_1 ($n \times m$) contains the $m = g - 1$ endogenous variables on the right-hand side;
- X_1 ($n \times k$) contains the k included exogenous variables (with the constant);
- X ($n \times K$) contains *all* exogenous variables of the system; X_2 contains the $K - k$ excluded ones.

In the demand equation (7.1): $y_1 = Q$, $Y_1 = P$, $X_1 = [\mathbf{1} \ I]$, $X = [\mathbf{1} \ I \ W]$ and $X_2 = W$.

OLS applied to (8.1) gives

$$\hat{\theta}_{OLS} = (D'D)^{-1}D'y_1 = \theta + \left(\frac{D'D}{n}\right)^{-1} \frac{D'u}{n}.$$

- The endogenous regressors Y_1 are correlated with u , so $\text{plim } D'u/n \neq 0$ and $\text{plim } \hat{\theta}_{OLS} \neq \theta$.
- This inconsistency is called **simultaneous equations bias**.

Example 8.1 (Price is correlated with the demand disturbance)

From the reduced form in Chapter 7,

$$P_t = \pi_{10} + \pi_{11}I_t + \pi_{12}W_t + (u_{Dt} - u_{St})/\Delta.$$

Since I_t and W_t are uncorrelated with u_{Dt} ,

$$\begin{aligned}\text{Cov}(P_t, u_{Dt}) &= \frac{1}{\Delta} \{ \text{Cov}(u_{Dt}, u_{Dt}) - \text{Cov}(u_{St}, u_{Dt}) \} \\ &= \frac{\sigma_D^2 - \sigma_{DS}}{\Delta}.\end{aligned}$$

Example 8.1 (cont.)

In the numerical market ($\Delta = 3$), if the two disturbances are uncorrelated and $\sigma_D^2 = 9$, then

$$\text{Cov}(P_t, u_{Dt}) = 9/3 = 3 \neq 0.$$

Economically: a positive demand shock raises both quantity demanded and the equilibrium price, so the price carries part of the demand shock.

- OLS is consistent for the **reduced-form equations**, because their right-hand sides contain only exogenous variables.
- It is also appropriate for a structural equation whose right-hand variables are all exogenous or predetermined (and uncorrelated with its disturbance).

Practice

- Exercise 8.1 (Exercises and Solutions booklet).

Next lecture

Lecture 55: Indirect least squares.