

Lecture 55: Indirect least squares

Econometrics ■ Unit IV: Simultaneous equations models

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Previous lecture

Lecture 54: OLS in a structural equation: inconsistency and where it is appropriate.

Today

- Indirect least squares
 - The method
 - ILS in the demand–supply model
 - Properties and limitation

Indirect least squares (ILS) applies to an **exactly identified** equation.

1. Derive the reduced form of the system.
2. Estimate each reduced-form equation by OLS (regress each endogenous variable on all exogenous variables).
3. Solve the relations between reduced-form and structural coefficients (found in the identification analysis) for the structural coefficients, using the estimated $\hat{\pi}$'s.

It is called “indirect” because the structural coefficients are obtained indirectly from OLS estimates of the reduced form.

From Chapter 7, write the estimated reduced forms as

$$\hat{P} = \hat{\pi}_{10} + \hat{\pi}_{11}I + \hat{\pi}_{12}W, \quad \hat{Q} = \hat{\pi}_{20} + \hat{\pi}_{21}I + \hat{\pi}_{22}W.$$

The recovery formulas are:

Demand (uses excluded W):

$$\hat{\alpha}_1 = \frac{\hat{\pi}_{22}}{\hat{\pi}_{12}}, \quad \hat{\alpha}_2 = \hat{\pi}_{21} - \hat{\alpha}_1 \hat{\pi}_{11}, \quad \hat{\alpha}_0 = \hat{\pi}_{20} - \hat{\alpha}_1 \hat{\pi}_{10};$$

Supply (uses excluded I):

$$\hat{\beta}_1 = \frac{\hat{\pi}_{21}}{\hat{\pi}_{11}}, \quad \hat{\beta}_2 = \hat{\pi}_{22} - \hat{\beta}_1 \hat{\pi}_{12}, \quad \hat{\beta}_0 = \hat{\pi}_{20} - \hat{\beta}_1 \hat{\pi}_{10}.$$

Example 8.2 (ILS)

Suppose OLS on the reduced form gives

$$\hat{P} = 6 + 0.2I + 0.4W, \quad \hat{Q} = 8 + 0.3I - 0.6W.$$

Demand.

$$\hat{\alpha}_1 = \frac{-0.6}{0.4} = -1.5,$$

$$\begin{aligned}\hat{\alpha}_2 &= 0.3 - (-1.5)(0.2) \\ &= 0.3 + 0.3 = 0.6,\end{aligned}$$

$$\begin{aligned}\hat{\alpha}_0 &= 8 - (-1.5)(6) \\ &= 8 + 9 = 17.\end{aligned}$$

Supply.

$$\hat{\beta}_1 = \frac{0.3}{0.2} = 1.5,$$

$$\begin{aligned}\hat{\beta}_2 &= -0.6 - 1.5(0.4) \\ &= -1.2,\end{aligned}$$

$$\hat{\beta}_0 = 8 - 1.5(6) = -1.$$

Example 8.2 (cont.)

So the estimated structural equations are

$$Q = 17 - 1.5P + 0.6I, \quad Q = -1 + 1.5P - 1.2W.$$

Check. Equating them:

$$17 - 1.5P + 0.6I = -1 + 1.5P - 1.2W,$$

so $3P = 18 + 0.6I + 1.2W$ and $P = 6 + 0.2I + 0.4W$. Substituting in demand:

$$\begin{aligned} Q &= 17 - 1.5(6 + 0.2I + 0.4W) + 0.6I \\ &= 17 - 9 - 0.3I - 0.6W + 0.6I = 8 + 0.3I - 0.6W. \end{aligned}$$

Both estimated reduced forms are reproduced.

- The OLS estimates $\hat{\pi}$ are consistent, and the ILS estimates are continuous functions of them (ratios with nonzero denominators), so ILS estimates are **consistent**.
- They are generally **biased** in small samples, since a ratio of unbiased estimates is not unbiased.
- ILS cannot be used directly for an **over-identified** equation. In the over-identified demand equation of Chapter 7, α_1 equals both π_{22}/π_{12} (from W) and π_{23}/π_{13} (from R), where π_{13}, π_{23} are the reduced-form coefficients of R .
- In a sample the two estimated ratios usually differ, say -1.5 and -1.8 , and ILS gives no rule for combining them. Two-stage least squares solves this problem.

Practice

- Exercise 8.2 (Exercises and Solutions booklet).

Next lecture

Lecture 56: Two-stage least squares: the method, derivation and consistency.