Lecture 60: Properties of LIML; comparison of the methods; summary

Econometrics ▪ Unit IV: Simultaneous equations models

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This lecture

Previous lecture

Lecture 59: LIML under exact identification; the k-class; a LIML calculation.

Today

- Limited information maximum likelihood
  - Properties
- Comparison of the methods
- Summary of key formulas

Properties of LIML

- LIML is consistent and, in large samples, has the same distribution as 2SLS.
- It does not depend on which endogenous variable is chosen for normalisation.
- Like all methods, it needs the equation to be identified and the excluded variables to be truly exogenous.

Exam tip

In the 2SLS derivation, show that P_X is symmetric and idempotent and that $P_X X_1 = X_1$. In LIML, explain why the *smallest* root of $|W_1 - \kappa W| = 0$ is chosen.

Comparison of the methods

Method	How it is computed	When it is used	Main property
OLS	Regress y_1 on Y_1 and X_1	Reduced forms; equations with only exogenous regressors	Inconsistent for structural equations with endogenous regressors
ILS	OLS on reduced form, then solve for structural coefficients	Exactly identified equations	Consistent; not usable for over-identified equations
2SLS	Replace Y_1 by $\widehat{Y}_1$ from all exogenous variables, then OLS	Exactly or over-identified equations	Consistent; equals ILS and IV when exactly identified
LIML	Minimise the variance ratio; smallest root of $\lvert W_1 - \kappa W \rvert = 0$	Exactly or over-identified equations	Consistent; equals 2SLS when exactly identified

Summary of key formulas

- Structural equation: $y_1 = Y_1\boldsymbol{\beta} + X_1\boldsymbol{\gamma} + u = D\boldsymbol{\theta} + u$; $X =$ all exogenous variables.
- OLS on a structural equation is inconsistent (simultaneous equations bias).
- ILS: OLS on the reduced form, then solve for structural coefficients; only for exactly identified equations.
- 2SLS: $\widehat{Y}_1 = P_X Y_1$, $\widehat{\boldsymbol{\theta}}_{2SLS} = (D' P_X D)^{-1} D' P_X y_1$, $P_X = X(X'X)^{-1}X'$; residuals use the original Y_1.
- Exactly identified: ILS $=$ IV $=$ 2SLS $=$ LIML.
- LIML: minimise $\kappa = \boldsymbol{a}' W_1 \boldsymbol{a} / \boldsymbol{a}' W \boldsymbol{a}$; smallest root of $|W_1 - \kappa W| = 0$; $\boldsymbol{a} = (1, -\boldsymbol{\beta}')'$.
- k-class: $\kappa = 0$ OLS, $\kappa = 1$ 2SLS, $\kappa = \widehat{\kappa}$ LIML.

After this lecture

Practice

- Exercise 8.6 (Exercises and Solutions booklet).
- R practical P13: OLS, ILS, 2SLS and LIML.

End of the course

This is the last lecture of Econometrics.