

Lecture 5: The minimax principle; minimax action in a no-data problem

Statistical Inference–II ■ Unit I: Decision Theory

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Previous lecture

Lecture 4: Uniformly best rules; admissibility; linear estimators of a normal mean.

Today

- The Minimax Principle
 - Worked Example: Minimax in a Finite Problem

The minimax principle

According to the minimax approach we guard against the worst possible outcome, requiring that the chosen decision rule provide maximum protection against the highest possible risk.

Definition 5.1 (Maximum risk; minimax rule)

The *maximum risk* of a rule δ is

$$\bar{R}(\delta) = \sup_{\theta \in \Theta} R(\theta, \delta).$$

A rule $\delta_M \in \mathcal{D}$ is called a *minimax rule* in $\mathcal{D}$ if it minimises the maximum risk, that is, if

$$\sup_{\theta \in \Theta} R(\theta, \delta_M) \leq \sup_{\theta \in \Theta} R(\theta, \delta) \quad \text{for every } \delta \in \mathcal{D}.$$

Equivalently, $\delta_M = \arg \min_{\delta \in \mathcal{D}} \sup_{\theta \in \Theta} R(\theta, \delta)$. The common value $\inf_{\delta} \sup_{\theta} R(\theta, \delta)$ is called the *minimax risk* of the problem.

The minimax principle criticised

Remark 5.2 (The minimax principle criticised)

The minimax approach has been called the *pessimist's* approach, since it is in the nature of a pessimist always to think of the worst and to act accordingly.

- It is not, however, true that one thereby necessarily acts wisely: a rule that guards against the worst may have comparatively very high risk at values of θ that are in fact far more plausible.
- The minimax criterion also ignores any prior information about θ and can be excessively conservative. End-of-chapter Exercise 1.10 exhibits a minimax rule that ignores the data entirely and is manifestly undesirable.

On the other hand, minimax rules have a genuine appeal in situations of complete prior ignorance, in adversarial contexts, and when a guaranteed performance bound is required (as in acceptance sampling or in the design of safety-critical systems).

Note 5.3 (A useful sufficient condition)

If a rule has *constant risk* over Θ and is admissible, then it is minimax. More useful in practice is the Bayes-based criterion established later (the corollary “Bayes rule with constant risk is minimax” in the section on relations among Bayes, minimax and admissible rules):

a Bayes rule with constant risk is minimax.

Example 5.4 (No-data minimax)

Let $\Theta = \{\theta_1, \theta_2\}$ and $\mathcal{A} = \{a_1, a_2\}$ with the loss function given in the following table. (Throughout these notes we adopt the convention that **rows are indexed by the action and columns by the state of nature.**)

$L(\theta, a)$	θ_1	θ_2
a_1	20	35
a_2	25	15

Find the minimax action.

Solution

No data are available, so an action is chosen directly. For each action we compute the maximum loss over Θ :

	θ_1	θ_2	$\max_{\theta} L(\theta, a)$
a_1	20	35	35
a_2	25	15	25

The minimum of the column of maxima is $\min\{35, 25\} = 25$, attained at a_2 . Hence a_2 is the minimax action among non-randomised actions, with maximum loss 25.

Note that neither action dominates the other (a_1 is better at θ_1 , a_2 at θ_2), so both a_1 and a_2 are admissible. The minimax criterion selects one of them, but admissibility alone does not.

Remark 5.5

A *randomised* rule, which chooses between a_1 and a_2 by a chance mechanism, can have a smaller maximum expected loss than 25.

This is worked out in “Randomised Minimax Rules in Finite Problems”, after Bayes rules and least favourable priors have been introduced.

Practice (Lecture Notes, Chapter 1)

- Short-answer questions (Section 1.12): Q7.

Next lecture

Lecture 6: Minimax and Bayes rules among non-randomised rules: the Bernoulli example.