

Lecture 21: Sequential procedures; stopping rules and stopping times

Statistical Inference–II ■ Units II–III: Sequential Analysis

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Units II–III: Sequential Analysis

Lecture 21 begins Units II–III: Sequential Analysis.

Today

- Introduction

Sequential analysis deals with statistical procedures in which the number of observations is not fixed in advance. After each observation, the accumulated information is examined and a decision is made either to stop sampling or to continue. Consequently, the final sample size is a positive integer-valued random variable.

A sequential procedure has two components.

- (i) A *stopping rule* specifies, after every stage, whether sampling is to stop or continue.
- (ii) An *action rule* specifies the inference to be reported when sampling stops, such as accepting one of two hypotheses or reporting an estimate.

Let $X_1, X_2, \dots$ be observations and let $\mathcal{F}_m = \sigma(X_1, \dots, X_m)$ denote the information available after m observations.

Definition 21.1 (Stopping time)

A random variable N taking values in $\{1, 2, \dots\} \cup \{\infty\}$ is a *stopping time* if $\{N = m\} \in \mathcal{F}_m$ for every m .

Thus the decision to stop at stage m depends only on observations already recorded. Equivalently,

$$\{N \geq i\} = \{N \leq i - 1\}^c \in \mathcal{F}_{i-1} \quad \text{for every } i;$$

this simple fact is used repeatedly below.

In a fixed-sample test, the sample size n is chosen before sampling and the sample space is divided into acceptance and rejection regions. In a sequential test, the sample size N is random and, at each stage, the accumulated sample space is divided into acceptance, continuation and rejection regions.

The principal advantage is that a sequential test often reaches a decision with a smaller expected sample size while controlling both Type I and Type II error probabilities.

The theory presented here is due to Wald (1947). Textbook accounts at the level of these notes are given by Goon, Gupta and Dasgupta (2003), Rohatgi and Saleh (2015) and Ferguson (1967, Ch. 7); the last places sequential testing within the decision-theoretic framework of Chapter 1.

Practice (Lecture Notes, Chapter 2)

- Short-answer questions (Section 2.12): Q1, Q2.

Next lecture

Lecture 22: Wald's SPRT; the log-likelihood ratio as a random walk.