

Lecture 30: The parametric method for drawing the O.C. curve

Statistical Inference–II ■ Units II–III: Sequential Analysis

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Previous lecture

Lecture 29: The O.C. function: the root $h(\theta)$ and the O.C. formula.

Today

- Operating Characteristic Function

Remark 30.1 (Consistency checks)

With $A = (1 - \beta)/\alpha$ and $B = \beta/(1 - \alpha)$:

(i) At $\theta = \theta_0$, $h = 1$ and

$$Q(\theta_0) \approx \frac{A - 1}{A - B} = \frac{(1 - \alpha - \beta)/\alpha}{(1 - \alpha - \beta)/\{\alpha(1 - \alpha)\}} = 1 - \alpha.$$

(ii) At $\theta = \theta_1$, $h = -1$ and

$$Q(\theta_1) \approx \frac{A^{-1} - 1}{A^{-1} - B^{-1}} = \frac{(\alpha + \beta - 1)/(1 - \beta)}{(\alpha + \beta - 1)/\{\beta(1 - \beta)\}} = \beta.$$

Consistency checks (cont.)

(iii) As $h \rightarrow 0$, $A^h = 1 + h \log A + o(h)$ and $B^h = 1 + h \log B + o(h)$, so

$$Q \longrightarrow \frac{\log A}{\log A - \log B},$$

which is the O.C. value at the zero-drift point (section on the Average Sample Number function).

The parametric method for drawing the O.C. curve

The parametric method for drawing the O.C. curve

Solving $\phi_\theta(h) = 1$ for h at each θ is usually awkward. Instead, solve it for θ as a function of h :

1. For a grid of values $h \in \{\dots, -2, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2, \dots\}$ compute $\theta(h)$ from $\phi_\theta(h) = 1$ (closed forms are given in the section on distribution-specific SPRTs); $h = 0$ is interpreted as the limit $h \rightarrow 0$, i.e. the zero-drift value θ^* .
2. Compute $Q(h) = (A^h - 1)/(A^h - B^h)$, with $Q(0) = \log A/(\log A - \log B)$.
3. Plot the points $(\theta(h), Q(h))$ and join them.

The values $h = 1$ and $h = -1$ always give the points $(\theta_0, 1 - \alpha)$ and (θ_1, β) .

Practice (Lecture Notes, Chapter 2)

- Exercises (Section 2.13): 2.6.

Next lecture

Lecture 31: The A.S.N. function.