

Lecture 32: The zero-drift case

Statistical Inference-II ■ Units II–III: Sequential Analysis

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M.Sc. Statistics, Semester III
Academic Session 2026–2027

Previous lecture

Lecture 31: The A.S.N. function.

Today

- Average Sample Number Function
 - The zero-drift case

At a parameter value θ^* for which $\mu_Z(\theta^*) = 0$, the preceding formula is of the indeterminate form $0/0$ and must not be used directly. In the zero-drift case Wald's second identity gives

$$\mathbb{E}_{\theta^*}(W_N^2) = \mathbb{E}_{\theta^*}(N)\mathbb{E}_{\theta^*}(Z_1^2).$$

With the boundary approximation and the zero-mean condition,

$$Q(\theta^*) \log B + [1 - Q(\theta^*)] \log A \approx 0,$$

so

$$Q(\theta^*) \approx \frac{\log A}{\log A - \log B}.$$

Moreover,

$$\mathbb{E}_{\theta^*}(W_N^2) \approx Q(\theta^*)(\log B)^2 + [1 - Q(\theta^*)](\log A)^2 = -(\log A)(\log B).$$

(Indeed, with $a = \log A$ and $b = \log B$,
 $\{ab^2 - ba^2\}/(a - b) = ab(b - a)/(a - b) = -ab$.) Hence the separate limiting approximation is

$$\mathbb{E}_{\theta^*}(N) \approx -\frac{(\log A)(\log B)}{\mathbb{E}_{\theta^*}(Z_1^2)}.$$

Since $\log A > 0$ and $\log B < 0$, this quantity is positive. The A.S.N. is typically largest near the zero-drift value because the observations there do not strongly favour either hypothesis.

Practice (Lecture Notes, Chapter 2)

- Short-answer questions (Section 2.12): Q11.

Next lecture

Lecture 33: SPRT for Bernoulli and binomial populations.