Lecture 34: SPRT for a Poisson population

Statistical Inference–II ▪ Units II–III: Sequential Analysis

Kapil Kumar

Department of Statistics
Chaudhary Charan Singh University, Meerut

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This lecture

Previous lecture

Lecture 33: SPRT for Bernoulli and binomial populations.

Today

- Distribution-Specific SPRTs
  - Poisson distribution

Poisson: likelihood ratio and lines

Let $X_i \sim \text{Poisson}(\lambda)$ and test $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1$, $\lambda_1 > \lambda_0$. With $T_m = \sum_{i=1}^{m} X_i$,

$$\Lambda_m = \exp\{-m(\lambda_1 - \lambda_0)\} \left(\frac{\lambda_1}{\lambda_0}\right)^{T_m},$$

$$W_m = T_m \log \frac{\lambda_1}{\lambda_0} - m(\lambda_1 - \lambda_0).$$

Lines.

$$\boxed{\frac{\log B + m(\lambda_1 - \lambda_0)}{\log(\lambda_1/\lambda_0)} < T_m < \frac{\log A + m(\lambda_1 - \lambda_0)}{\log(\lambda_1/\lambda_0)}.}$$

Poisson: O.C. function

Since $\mathbb{E}_\lambda(c^X) = e^{\lambda(c-1)}$,

$$\phi_\lambda(h) = e^{-h(\lambda_1-\lambda_0)} \exp\left[\lambda\left\{\left(\frac{\lambda_1}{\lambda_0}\right)^h - 1\right\}\right] = 1$$

$$\iff \lambda\left\{\left(\frac{\lambda_1}{\lambda_0}\right)^h - 1\right\} = h(\lambda_1 - \lambda_0),$$

so that

$$\boxed{\lambda(h) = \frac{h(\lambda_1 - \lambda_0)}{(\lambda_1/\lambda_0)^h - 1}, \qquad h \neq 0.}$$

Indeed $\lambda(1) = \lambda_0$ and $\lambda(-1) = \lambda_1$.

Poisson: A.S.N. function

$$\mu_Z(\lambda) = \lambda \log \frac{\lambda_1}{\lambda_0} - (\lambda_1 - \lambda_0), \qquad \lambda^* = \frac{\lambda_1 - \lambda_0}{\log(\lambda_1/\lambda_0)} = \lim_{h \to 0} \lambda(h),$$

$$\mathbb{E}_{\lambda^*}(Z_1^2) = \mathrm{Var}_{\lambda^*}(Z_1) = \lambda^* \left(\log \frac{\lambda_1}{\lambda_0} \right)^2,$$

$$\mathbb{E}_{\lambda^*}(N) \approx -\frac{(\log A)(\log B)}{\lambda^* \{\log(\lambda_1/\lambda_0)\}^2}.$$

After this lecture

Practice (Lecture Notes, Chapter 2)

- Exercises (Section 2.13): 2.3.

Next lecture

Lecture 35: SPRT for a normal mean and a normal variance.