

Lecture 39: Solved numerical examples: exponential and gamma

Statistical Inference–II ■ Units II–III: Sequential Analysis

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Previous lecture

Lecture 38: Solved numerical examples: binomial, Poisson and normal.

Today

- Solved Numerical Examples

Example 39.1 (Exponential life testing)

Lifetimes (in hours) of a component are exponential with rate θ . Test $H_0 : \theta = 0.01$ (mean life 100 hours) against $H_1 : \theta = 0.02$ (mean life 50 hours) with $\alpha = 0.05$, $\beta = 0.10$, using successive lifetimes

35, 62, 18, 41, 27, 55, 12, 30, 20.

Solution

Here $\log(\theta_1/\theta_0) = \log 2 = 0.6931$ and $\theta_1 - \theta_0 = 0.01$. By the exponential SPRT lines,

reject H_0 if $S_m \leq 69.31 m - 289.04$,
accept H_0 if $S_m \geq 69.31 m + 225.13$.

Example: Exponential life testing (solution, cont.)

Solution (cont.)

m	1	2	3	4	5
S_m	35	97	115	156	183
rejection line	-219.7	-150.4	-81.1	-11.8	57.5
acceptance line	294.4	363.8	433.1	502.4	571.7

m	6	7	8	9
S_m	238	250	280	300
rejection line	126.9	196.2	265.5	334.8
acceptance line	641.0	710.3	779.6	849.0

The total lifetime first falls on or below the rejection line at $m = 9$ ($300 \leq 334.8$), so the test stops at $N = 9$ and concludes that the mean life is 50 rather than 100 hours. (Check:

$$W_9 = 9 \log 2 - 0.01(300) = 3.2383 > \log A.)$$

Example: Exponential life testing (solution, cont.)

Solution (cont.)

O.C. and A.S.N. $\mu_Z(0.01) = \log 2 - 1 = -0.3069$ and
 $\mu_Z(0.02) = \log 2 - 0.5 = 0.1931$, so

$$\mathbb{E}_{0.01}(N) \approx \frac{-1.9942}{-0.3069} = 6.50, \quad \mathbb{E}_{0.02}(N) \approx \frac{2.3762}{0.1931} = 12.30.$$

The zero-drift rate is $\theta^* = 0.01/\log 2 = 0.014427$ (mean life 69.3 hours),
where $Q \approx 0.5621$ and $\mathbb{E}_{\theta^*}(N) \approx 6.5071/(\log 2)^2 = 13.54$.

Example 39.2 (Gamma SPRT with known shape)

Observations are gamma with known shape $k = 2$ and unknown rate θ . Test $H_0 : \theta = 1$ against $H_1 : \theta = 2$ with $\alpha = 0.05$, $\beta = 0.10$, using the observations 2.4, 1.1, 3.0.

Solution

With $k \log(\theta_1/\theta_0) = 2 \log 2 = 1.3863$ and $\theta_1 - \theta_0 = 1$, the gamma SPRT lines give

reject H_0 if $S_m \leq 1.3863 m - 2.8904$,

accept H_0 if $S_m \geq 1.3863 m + 2.2513$.

Example: Gamma SPRT with known shape (solution, cont.)

Solution (cont.)

m	1	2	3
S_m	2.4	3.5	6.5
rejection line	-1.504	-0.118	1.269
acceptance line	3.638	5.024	6.410

At $m = 3$, $S_3 = 6.5 \geq 6.410$, so the test stops at $N = 3$ and accepts H_0 .
(Check: $W_3 = 3(1.3863) - 6.5 = -2.3411 < \log B$.)

Example: Gamma SPRT with known shape (solution, cont.)

Solution (cont.)

O.C. and A.S.N. The O.C. curve is that of the exponential SPRT for $\theta_0 = 1$, $\theta_1 = 2$. The drifts are

$$\mu_Z(1) = 2(\log 2 - 1) = -0.6137, \quad \mu_Z(2) = 2(\log 2 - 0.5) = 0.3863,$$

so $\mathbb{E}_1(N) \approx 3.25$ and $\mathbb{E}_2(N) \approx 6.15$; at $\theta^* = 1/\log 2 = 1.4427$,

$$\mathbb{E}_{\theta^*}(N) \approx 6.5071/\{2(\log 2)^2\} = 6.77.$$

These are exactly half of the corresponding values 6.50, 12.30 and 13.54 of the exponential life-testing example, as the gamma-SPRT discussion predicts.

Example: Gamma SPRT with known shape (solution, concluded)

Solution (cont.)

That example has the same ratio $\theta_1/\theta_0 = 2$, and the A.S.N. values at θ_0 , θ_1 and θ^* depend on the rates only through this ratio, for example

$$\mu_Z(\theta_0) = k\{\log(\theta_1/\theta_0) - (\theta_1/\theta_0 - 1)\}.$$

Practice (Lecture Notes, Chapter 2)

- Exercises (Section 2.13): 2.4.

Next lecture

Lecture 40: Summary and revision.