Lecture 46: One-sided bounds; duality with the sign test; principles of nonparametric testing

Statistical Inference–II ▪ Units IV–V: Nonparametric Inference

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This lecture

Previous lecture

Lecture 45: Distribution-free confidence intervals for quantiles.

Today

- Distribution-Free Confidence Intervals for Population Quantiles
  - One-Sided Intervals
  - Duality with the Sign Test
- General Principles of Nonparametric Testing
  - Exact and Asymptotic Tests
  - One-Sided and Two-Sided Alternatives
  - Ties and Zero Differences
  - Continuity Correction

One-sided intervals

Using the conventions $X_{(0)} = -\infty$ and $X_{(n+1)} = \infty$, one-sided bounds are obtained similarly. For example,

$$\mathbb{P}\{\xi_p \leq X_{(s)}\} = \mathbb{P}(Y \leq s-1) = \sum_{j=0}^{s-1} \binom{n}{j} p^j (1-p)^{n-j}.$$

The smallest s for which this probability is at least $1-\alpha$ gives an upper confidence bound.

Duality with the sign test

An exact confidence interval for a median may also be obtained by inverting the two-sided sign test.

Values of η_0 that are not rejected by the test

$$H_0 : \eta = \eta_0$$

form the confidence set.

This duality explains why the same binomial probabilities appear in both procedures.

Exact and asymptotic tests

A nonparametric test begins with a statistic whose null distribution is obtained by symmetry, exchangeability, random labelling, signs, ranks or the probability integral transformation.

An *exact test* calculates a tail probability under the finite-sample null distribution.

An *asymptotic test* replaces that distribution by a limiting normal, chi-square or Kolmogorov distribution.

Exact tests are preferable for small samples, but large-sample approximations are convenient when enumeration is difficult.

One-sided and two-sided alternatives

The rejection region must correspond to the scientific alternative.

If large values of a statistic support one direction and small values support the opposite direction, a two-sided p-value is not always exactly twice the smaller one-sided probability in a discrete or asymmetric null distribution.

The adopted convention should be stated.

Ties and zero differences

Classical rank-test derivations usually assume a continuous distribution, so ties have probability zero.

Real data may contain ties because of rounding or a discrete measurement scale. Average ranks and variance corrections are then used.

In paired procedures, zero differences are commonly removed before the sample size and ranks are computed.

Continuity correction

When a discrete statistic is approximated by a normal distribution, a continuity correction of $1/2$ is often used.

- For a lower-tail calculation, T is replaced by $T + 1/2$;
- for an upper-tail calculation it is replaced by $T - 1/2$.

After this lecture

Practice (Lecture Notes, Chapter 3)

- Short-answer questions (Section 3.14): Q15.
- Exercises (Section 3.15): 3.6.

Next lecture

Lecture 47: Runs: the null distribution of the number of runs.