

Lecture 50: One-sample sign test

Statistical Inference–II ■ Units IV–V: Nonparametric Inference

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This lecture

Previous lecture

Lecture 49: Runs up and down; the Wald–Wolfowitz two-sample runs test.

Today

- Sign Tests for One and Two Samples
 - One-Sample Sign Test for a Median

The sign test uses only whether each observation lies above or below a hypothesised median.

It is valid without symmetry and is robust, but it ignores the magnitudes of the deviations.

One-sample sign test for a median

Let $X_1, \dots, X_n$ be independent observations from a continuous population with median η . To test

$$H_0 : \eta = \eta_0,$$

define

$$S_+ = \sum_{i=1}^n \mathbf{1}(X_i > \eta_0), \quad S_- = \sum_{i=1}^n \mathbf{1}(X_i < \eta_0).$$

Observations equal to η_0 are omitted, and $n^* = S_+ + S_-$ is the effective sample size. Under H_0 ,

$$S_+ \sim \text{Bin}(n^*, 1/2).$$

- For $H_1 : \eta > \eta_0$, reject for large S_+ .
- For $H_1 : \eta < \eta_0$, reject for small S_+ .
- For a two-sided alternative, reject when S_+ is too close to 0 or n^* .

For large n^* ,

$$Z = \frac{S_+ - n^*/2}{\sqrt{n^*/4}}$$

is approximately standard normal, with continuity correction.

Exact critical values for $n^* \leq 35$: Tables in Appendix B of the lecture notes. For a one-sided test compare the count that is small under H_1 (S_- when $H_1 : \eta > \eta_0$, S_+ when $H_1 : \eta < \eta_0$); for a two-sided test compare $\min(S_+, S_-)$.

Example: one-sample sign test

Example 50.1

Suppose that, after deleting zero deviations from η_0 , there are $n^* = 12$ observations, of which 10 are above η_0 and 2 below. Test $H_0 : \eta = \eta_0$ against a two-sided alternative.

Solution

Here $S_+ = 10$ and, under H_0 , $S_+ \sim \text{Bin}(12, \frac{1}{2})$, which is symmetric about 6. The exact two-sided probability is

$$2\mathbb{P}\{\text{Bin}(12, 1/2) \leq 2\} = 2 \sum_{j=0}^2 \binom{12}{j} 2^{-12} = 0.0386.$$

Thus the null median is rejected at the 5% level against a two-sided alternative.

Practice (Lecture Notes, Chapter 3)

- Short-answer questions (Section 3.14): Q7.
- Exercises (Section 3.15): 3.12.

Next lecture

Lecture 51: Paired sign test; confidence interval for a median.