

Lecture 51: Paired sign test; confidence interval for a median

Statistical Inference–II ■ Units IV–V: Nonparametric Inference

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Previous lecture

Lecture 50: One-sample sign test.

Today

- Sign Tests for One and Two Samples
 - Paired-Sample Sign Test
 - Confidence Interval for a Median
 - Assumptions and Interpretation

For matched pairs (X_i, Y_i) , define

$$D_i = Y_i - X_i.$$

Testing that the median paired difference is zero is exactly the one-sample sign test applied to the nonzero D_i 's.

Pairing must be respected: treating the two marginal samples as independent discards the design and gives an incorrect null distribution.

Example: paired sign test

Example 51.1 (Paired sign test)

Systolic blood pressure (mm Hg) of 12 patients was measured before and after a month of treatment with a new drug:

Patient	1	2	3	4	5	6	7	8	9	10	11	12
Before	148	152	139	160	145	150	155	138	162	141	149	157
After	140	147	142	148	145	145	148	140	152	138	143	148
$D = \text{Before} - \text{After}$	8	5	-3	12	0	5	7	-2	10	3	6	9

Test, at the 5% level, whether the drug reduces blood pressure.

Example: paired sign test (cont.)

Solution

Here $D = \text{Before} - \text{After}$, so that a positive D is a reduction. (Taking $D = \text{After} - \text{Before}$ instead, as in $D_i = Y_i - X_i$ above, would only interchange S_+ and S_- .) Let η_D be the median of D . Test $H_0 : \eta_D = 0$ against $H_1 : \eta_D > 0$.

Patient 5 has $D = 0$ and is deleted, so $n^* = 11$, with $S_+ = 9$ positive and $S_- = 2$ negative differences. Under H_0 , $S_- \sim \text{Bin}(11, \frac{1}{2})$, and small values of S_- support H_1 :

$$\mathbb{P}(S_- \leq 2) = \frac{\binom{11}{0} + \binom{11}{1} + \binom{11}{2}}{2^{11}} = \frac{1 + 11 + 55}{2048} = \frac{67}{2048} = 0.0327.$$

Example: paired sign test (cont.)

Solution (cont.)

Since $0.0327 < 0.05$, H_0 is rejected: the drug reduces blood pressure. (From the sign-test table in Appendix B of the lecture notes, the one-sided 5% critical value for $n^* = 11$ is 2, and $S_- = 2$ does not exceed it.)

The normal approximation with continuity correction gives

$$z = \frac{2 + 0.5 - 5.5}{\sqrt{11/4}} = -1.809, \quad \Phi(-1.809) = 0.035,$$

close to the exact value.

At the 1% level the sign test would not reject; the signed-rank test of the example “Paired signed-rank test with ties” uses the same data more efficiently.

The exact interval $[X_{(r)}, X_{(s)}]$ with $s = n + 1 - r$ developed in the Theorem on the exact confidence interval for a quantile is the confidence interval obtained by inverting the two-sided sign test (Exercise 3.14).

It is therefore natural to report a median estimate together with this interval rather than only a p -value.

The sign test assumes independent observations and a common continuous population distribution.

- It tests a median statement.
- It does not test a mean, and it does not require symmetry.
- Its power may be lower than that of a signed-rank or t procedure when their stronger assumptions are correct.

Practice (Lecture Notes, Chapter 3)

- Exercises (Section 3.15): 3.13, 3.14.

Next lecture

Lecture 52: Median test: construction; a k -sample example.