

SEM: Concepts and Applications

Structural equation modelling

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Plan of the session

1. Why SEM?
2. Path analysis
3. Measurement: confirmatory factor analysis
4. Estimation and fit, with a live CFA
5. Full SEM and mediation
6. Good practice and reporting

By the end you can read and draw a path diagram, trace direct and indirect effects, count degrees of freedom, judge a model by its fit indices, reliability and validity, test a mediated effect with a bootstrap interval, and report an SEM honestly.

PART 1

Why SEM?

Quantities without a ruler

- Job satisfaction, customer loyalty, depression, research anxiety, quality of life: none can be read from a dial
- Yet they drive decisions in education, health, business and policy
- We can only ask *several questions*, each a noisy, partial reflection of the construct

Customer loyalty, measured indirectly “Would you recommend us to a friend?”
“How likely are you to buy from us again?”
“Do you feel attached to this brand?”

SEM reasons *backwards* from such answers to the latent construct, and in the same model relates that construct to others.

What makes SEM different

SEM does two jobs at once:

1. builds an invisible **construct** from several measured indicators;
2. lets constructs **predict and explain** one another.

Regression	observed variables only
Factor analysis	latent variables, no paths
SEM	both, in one model

The one-sentence summary A structural equation model is good when the covariance pattern it *implies* is close to the covariance pattern *observed* in the data.

Path analysis, CFA, mediation and full SEM are all versions of this one idea.

Three traditions, one framework

1. **Factor analysis** (Spearman): test scores reflect an unobserved common factor $\Rightarrow$ *loadings*
2. **Path analysis** (Sewall Wright): arrow diagrams of directed effects $\Rightarrow$ *direct, indirect, total effects*
3. **Simultaneous equations** (econometrics): systems of related equations $\Rightarrow$ “*structural*”

Fusion Jöreskog and others joined them in the covariance-structure framework (LISREL). Today: LISREL, Mplus, AMOS and the open-source `lavaan` package in R.

The vocabulary is a museum of these three origins.

What SEM can do

Task	Question answered	Typical output
Path analysis	direct, indirect effects among observed variables?	path coefficients
CFA	do the items measure the constructs as theorised?	loadings
Full SEM	constructs related, net of measurement error?	structural paths
Mediation	does an effect pass through a mediator?	indirect, total effects
Invariance	is a construct measured alike across groups?	nested model tests
Multi-group SEM	do paths differ across groups?	group-specific paths
Latent growth	how do individuals change over time?	intercept, slope factors

Not a causality machine A tidy diagram is not proof of causation: the arrows encode the researcher's assumptions, and the data cannot certify every direction.

The covariance matrix is the target

$$s_{XY} = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$$

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}$$

$\mathbf{S} = (s_{jk})_{p \times p}$: $p(p+1)/2$ distinct entries

The model implies a covariance matrix $\Sigma(\theta)$ from a few parameters (loadings, paths, variances, covariances); estimation chooses θ so that $\Sigma(\theta) \approx \mathbf{S}$.

Units matter Covariance keeps the units needed for unstandardised estimates; correlation is unit-free.

Fit covariances A correlation matrix fitted as if it were a covariance matrix can give wrong SEs and χ^2 . Fit covariances; then ask for the standardised solution.

Regressions wired together

$$Y = a + b_1X_1 + \dots + b_kX_k + e$$

$$\beta^{\text{std}} = b \frac{s_X}{s_Y}$$

SEM is many regressions estimated *simultaneously*, some of whose variables are *latent*.

- **Unstandardised:** change in Y 's units per unit of X ; real-world meaning
- **Standardised:** change in SDs of Y per SD of X ; compares paths

Significant is not important With a large sample a tiny standardised path can be significant. Read size, uncertainty and meaning together.

The lavaan model language

```
# install once  
# install.packages("lavaan")  
library(lavaan)
```

lavaan (Rosseel, 2012) fits path models, CFA, full SEM and mediation with `sem()` and `cfa()`.

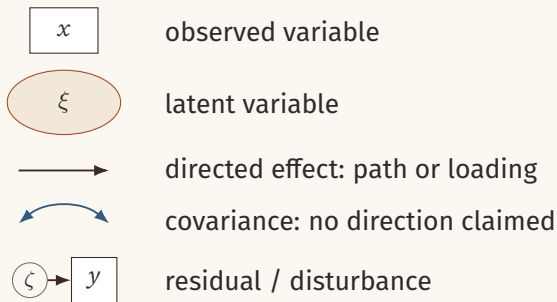
In words	Operator, example
factor measured by items	<code>=~</code> $F = x_1 + x_2 + x_3$
is regressed on	<code>~</code> $Y \sim X + M$
covariance	<code>~~</code> $X \sim M$
label a path	<code>*</code> $M \sim aX$
derived quantity	<code>:=</code> <code>ind</code> <code>:= a*b</code>

The diagram you draw is, almost literally, the model you type.

PART 2

Path analysis

Drawing conventions: the diagram is the model



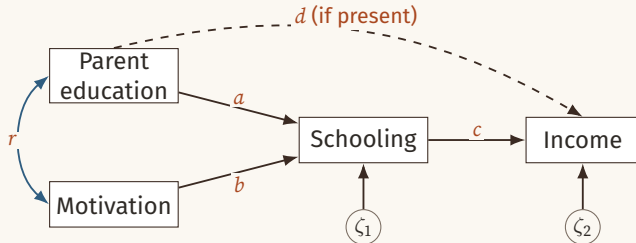
Two kinds of variable

Exogenous: no incoming one-headed arrow.

Endogenous: at least one; it always carries a disturbance.

Every arrow present is a parameter to estimate; every arrow *absent* is a claim (a zero) that the data can contradict.

A first path model: schooling and income



Effects of parent education on income:

direct: d

indirect (via schooling): $a c$

total: $d + a c$

Multiply coefficients along a directed route; add across routes.

Wright's tracing rules: a worked example

A legal route never goes forward and then backward; passes through at most one two-headed arrow; never visits a variable twice.

A model-implied correlation is the sum, over all legal routes, of the products of the coefficients along each route. Standardised values: $a = 0.45$, $b = 0.50$, $c = 0.65$, $r = 0.35$; no direct path d .

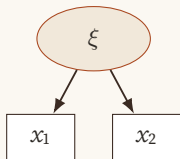
$$\begin{aligned}\text{Cor}(P, S) &= a + r b = 0.45 + 0.35 \times 0.50 \\ &= 0.625\end{aligned}$$

$$\begin{aligned}\text{Cor}(P, I) &= (a + r b) c = 0.625 \times 0.65 \\ &= 0.406\end{aligned}$$

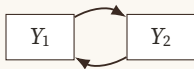
P parent education, S schooling, I income; a is the direct route, $r b$ the route through motivation.

If 0.406 is close to the *observed* correlation, this part of the model fits: the principle $\Sigma(\theta) \approx \mathbf{S}$ by hand.

Diagrams that cannot be estimated



(a) a factor with two indicators



(b) a reciprocal loop



(c) no disturbance on Y

- (a) is under-identified on its own: use at least three indicators per factor
- (b) is not identified from cross-sectional data without extra constraints (e.g. instruments)
- (c) claims $R^2 = 1$: every endogenous variable needs a disturbance

PART 3

Measurement: confirmatory factor analysis

The measurement model

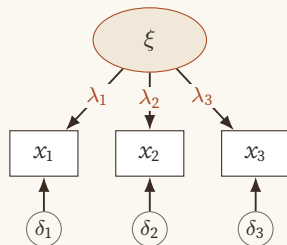
$$x_j = \lambda_j \xi + \delta_j, \quad j = 1, \dots, p$$

$$\text{Cov}(x_j, x_k) = \lambda_j \lambda_k \phi \quad (j \neq k)$$

$$\text{Var}(x_j) = \lambda_j^2 \phi + \theta_j$$

$\text{Var}(\xi) = \phi$, $\text{Var}(\delta_j) = \theta_j$; errors uncorrelated with each other and with ξ .

Items correlate *because they share a hidden cause*; what they do not share is item-specific error.



A high loading is a sharp thermometer for the construct; a low one is noisy.

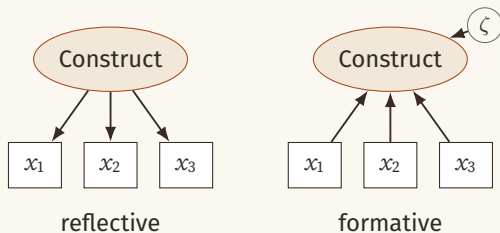
Confirmatory versus exploratory factor analysis

EFA discovers structure every item may load on every factor; you inspect how many factors emerge; *hypothesis-generating*.

CFA tests structure you declare which items load on which factor and fix the other loadings at 0; the model can misfit; *hypothesis-testing*.

EFA asks “what is the structure?”; CFA asks “is my proposed structure supported?”. SEM builds on CFA: you commit to a measurement model before fitting it. Running EFA and CFA on the *same* sample turns a confirmatory test into an exploratory one.

Reflective or formative?



- **Reflective:** the construct causes interchangeable items (depression $\rightarrow$ poor sleep, low appetite); items correlate
- **Formative:** items define the construct (SES from income, education, occupation); need not correlate, and ω does not apply

Decide on theory Treating a formative construct as reflective (or the reverse) is a specification error no fit index will flag.

Identification and degrees of freedom

A latent factor has no unit. Give it one by

- **marker:** fix one loading to 1 (`lavaan` default), or
- **fixed variance:** $\text{Var}(\xi) = 1$ (`std.lv = TRUE`).

Both give the same fit.

$$\text{df} = \frac{p(p+1)}{2} - q$$

p observed variables, q free parameters.

$\text{df} < 0$	under-identified: no unique estimates
$\text{df} = 0$	just-identified: fits perfectly, untestable
$\text{df} > 0$	over-identified: can misfit, so testable

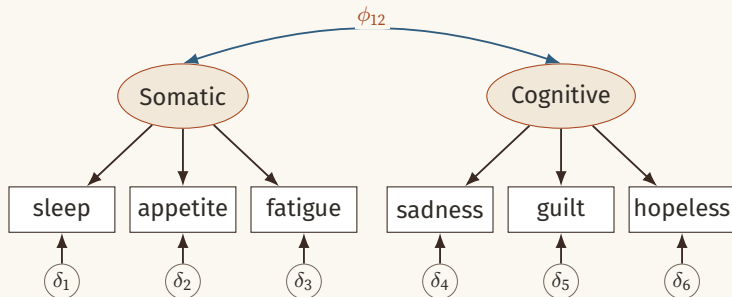
Necessary, not sufficient $\text{df} \geq 0$
does not guarantee identification: a factor with too few or weak indicators can still be empirically unidentified.

Counting degrees of freedom

Model	p	$p(p + 1)/2$	free parameters q	df
one factor, 3 items	3	6	3 loadings + 3 errors = 6	0
one factor, 5 items (research self-efficacy)	5	15	5 + 5 = 10	5
two correlated factors, 3 items each	6	21	6 + 6 + 1 covariance = 13	8
three correlated factors, 3 items each	9	45	9 + 9 + 3 covariances = 21	24

Reading it A one-factor model with three items reproduces **S** exactly, so its perfect “fit” is no evidence. The last row is the model we fit live in a few minutes.

Application: a depression screening scale



One distress factor or two related ones? The two-factor CFA ($df = 8$) needs sizeable loadings, a factor correlation well below 1, acceptable fit and reliability.

PART 4

Estimation and fit

Maximum likelihood and the χ^2 test

$$F_{\text{ML}}(\boldsymbol{\theta}) = \log |\boldsymbol{\Sigma}(\boldsymbol{\theta})| + \text{tr}\{\mathbf{S} \boldsymbol{\Sigma}(\boldsymbol{\theta})^{-1}\} - \log |\mathbf{S}| - p$$

$$T = (N - 1) F_{\text{ML}}(\hat{\boldsymbol{\theta}}) \approx \chi^2_{\text{df}}$$

H_0 : the model holds in the population, $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}(\boldsymbol{\theta})$.

$F_{\text{ML}} = 0$ only when $\boldsymbol{\Sigma}(\hat{\boldsymbol{\theta}}) = \mathbf{S}$. lavaan's default uses N in place of $N - 1$.

A welcome non-significance

Here a *large* p -value is good news: observed and implied covariances do not differ detectably.

But With large N trivial misfit becomes “significant”. SEM is never judged by χ^2 alone.

Approximate fit indices

$$\text{RMSEA} = \sqrt{\frac{\max(T - \text{df}, 0)}{\text{df} (N - 1)}}$$

$$\text{CFI} = 1 - \frac{\max(T - \text{df}, 0)}{\max(T_B - \text{df}_B, T - \text{df}, 0)}$$

$$\text{TLI} = \frac{T_B/\text{df}_B - T/\text{df}}{T_B/\text{df}_B - 1}$$

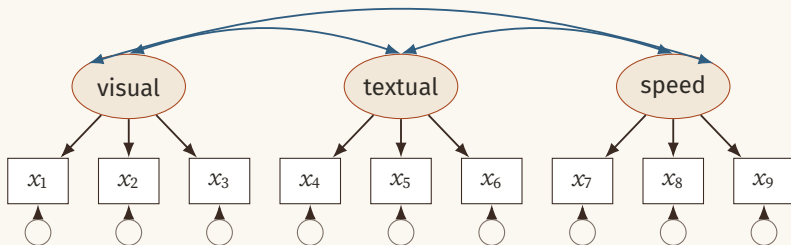
$$\text{SRMR} = \sqrt{\frac{\sum_{j \leq k} (r_{jk} - \hat{r}_{jk})^2}{p(p+1)/2}}$$

Index	Guide
CFI, TLI	≥ 0.95
RMSEA	≤ 0.06 , with 90% CI
SRMR	≤ 0.08

T_B, df_B : baseline (independence) model; guides from Hu and Bentler (1999).

Guides, not laws Read them as a panel; RMSEA is unstable when df is small.

Case study: Holzinger and Swineford (1939)



Classic data shipped with `lavaan`: nine mental-ability tests of 301 children in two schools; no missing values.

Visual: x_1 – x_3 (visual perception, cubes, lozenges); textual: x_4 – x_6 (paragraphs, sentences, word meaning); speed: x_7 – x_9 (addition, counting dots, capitals).

Fitting the three-factor CFA

```
hs_model <- 'visual  =~ x1 + x2 + x3
             textual =~ x4 + x5 + x6
             speed   =~ x7 + x8 + x9'
fit <- cfa(hs_model, data = HolzingerSwineford1939)
lavInspect(fit, "converged")

## [1] TRUE

round(fitMeasures(fit, c("chisq", "df", "pvalue", "cfi", "tli", "rmsea",
                        "rmsea.ci.lower", "rmsea.ci.upper", "srmr")), 3)

##          chisq          df          pvalue          cfi          tli
##      85.306      24.000          0.000          0.931          0.896
##      rmsea rmsea.ci.lower rmsea.ci.upper          srmr
##      0.092          0.071          0.114          0.065
```

Reading the fit, and checking the formulas

```
fm <- fitMeasures(fit, c("chisq", "df",  
  "baseline.chisq", "baseline.df"))  
N <- nobs(fit)  
rmsea <- sqrt(max(fm[["chisq"]] - fm[["df"]], 0) /  
  (fm[["df"]] * N)) # lavaan uses N  
cfi <- 1 - (fm[["chisq"]] - fm[["df"]]) /  
  (fm[["baseline.chisq"]] - fm[["baseline.df"]])  
round(c(RMSEA = rmsea, CFI = cfi), 3)  
  
## RMSEA    CFI  
## 0.092 0.931
```

Verdict: mediocre fit

$\chi^2(24) = 85.3, p < 0.001$; CFI = 0.931 and TLI = 0.896 fall short of 0.95; RMSEA = 0.092, 90% CI 0.071 to 0.114, is above 0.06; only SRMR = 0.065 is acceptable.

df = 45 – 21 = 24, as counted by hand.

Where is the misfit? Modification indices

```
mi <- modindices(fit, sort. = TRUE)
head(mi[, c("lhs", "op", "rhs", "mi", "epc")], 4)
```

##	lhs	op	rhs	mi	epc
## 30	visual	=~	x9	36.41	0.577
## 76	x7	~~	x8	34.15	0.536
## 28	visual	=~	x7	18.63	-0.422
## 78	x8	~~	x9	14.95	-0.423

mi: expected drop in χ^2 (1 df) if that fixed parameter were freed; epc: its expected value.

- Largest: x_9 (speeded discrimination of capitals) may also draw on *visual* ability, a plausible cross-loading
- x_7 and x_8 share extra covariance: two numerical speed tasks

Easily abused Free a parameter only if theory supports it, report it as a post-hoc change, and confirm it on new data.

Standardised loadings and factor correlations

```
std <- standardizedSolution(fit)
lam <- std[std$op == "~", ]
setNames(round(lam$est.std, 2), lam$rhs)

##      x1      x2      x3      x4      x5      x6      x7      x8      x9
## 0.77 0.42 0.58 0.85 0.86 0.84 0.57 0.72 0.67

round(unclass(lavInspect(fit, "cor.lv")), 2)

##           visual textual speed
## visual      1.00      0.46 0.47
## textual      0.46      1.00 0.28
## speed        0.47      0.28 1.00
```

- Textual items are sharp indicators (0.84 to 0.86)
- x_2 (cubes) is weak: 0.42, below the usual 0.50
- Factor correlations 0.28 to 0.47: related but distinct abilities

Reliability and validity of a construct

On the standardised solution, $\theta_j = 1 - \lambda_j^2$:

$$CR = \omega = \frac{(\sum_j \lambda_j)^2}{(\sum_j \lambda_j)^2 + \sum_j \theta_j}$$
$$AVE = \frac{\sum_j \lambda_j^2}{\sum_j \lambda_j^2 + \sum_j \theta_j} = \frac{1}{p} \sum_j \lambda_j^2$$

Fornell-Larcker: factors are distinct if each $\sqrt{AVE}$ exceeds that factor's correlations with the others.

Quantity	Acceptable	Good
std. loading	≥ 0.50	≥ 0.70
CR (ω)	≥ 0.70	≥ 0.80
AVE	≥ 0.50	
items per factor	3	4 or more

Conventions A loading of 0.48 does not sink a scale, and 0.72 does not rescue a poor theory.

Reliability and validity, computed

```
CR <- tapply(lam$est.std, lam$lhs, function(l)
  sum(l)^2 / (sum(l)^2 + sum(1 - l^2)))
AVE <- tapply(lam$est.std, lam$lhs,
  function(l) mean(l^2))
round(rbind(CR, AVE, sqrt_AVE = sqrt(AVE)), 2)

##           speed textual visual
## CR           0.69     0.89   0.63
## AVE           0.43     0.72   0.37
## sqrt_AVE      0.66     0.85   0.61
```

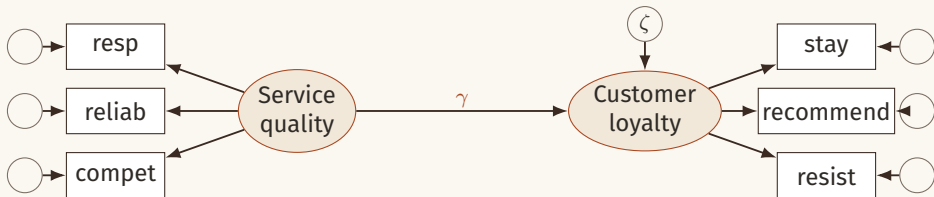
Reading it *Textual* is reliable (CR 0.89, AVE 0.72). *Visual* and *speed* have AVE below 0.50 (0.37, 0.43): weak convergent validity. Each $\sqrt{\text{AVE}}$ exceeds every factor correlation (largest 0.47), so discriminant validity holds.

Fit, reliability and validity are separate questions: a model can pass one and fail another.

PART 5

Full SEM and mediation

Full SEM: measurement and structure together



The structural path γ links the constructs *after* separating common variance from item error; a sum score keeps all the error.

Two-step approach Fit the measurement model (CFA) first; add structural paths only once it is sound (Anderson and Gerbing, 1988).

Measurement error weakens observed relationships

$$r_{\text{observed}} = r_{\text{true}} \sqrt{\rho_X \rho_Y}$$

Example: reliabilities $\rho_X = \rho_Y = 0.70$ and an observed sum-score correlation of 0.45.

$$r_{\text{true}} = \frac{0.45}{\sqrt{0.70 \times 0.70}} = \frac{0.45}{0.70} \approx 0.64$$

Disattenuation The construct-level relationship is stronger than the sum-score correlation suggests. SEM makes this correction automatically by modelling the error.

A telecom asking whether service quality drives loyalty wants the *construct*-level answer, not the attenuated one.

LISREL notation

Structural: $\eta = \mathbf{B}\eta + \mathbf{\Gamma}\xi + \zeta$

Measurement: $\mathbf{y} = \mathbf{\Lambda}_y\eta + \varepsilon, \quad \mathbf{x} = \mathbf{\Lambda}_x\xi + \delta$

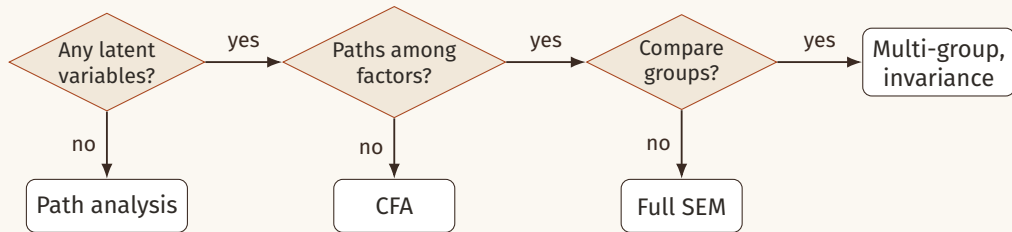
CFA part: $\Sigma_{xx}(\theta) = \mathbf{\Lambda}_x\mathbf{\Phi}\mathbf{\Lambda}_x^\top + \mathbf{\Theta}_\delta$

R^2 of an endogenous factor with a single

predictor: $R_\eta^2 = \frac{\gamma^2\phi}{\gamma^2\phi + \psi}, \psi = \text{Var}(\zeta).$

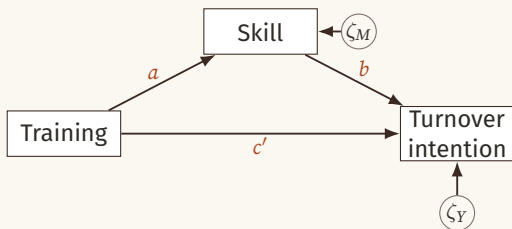
ξ, η	exogenous, endogenous
x, y	their indicators
$\mathbf{\Lambda}_x, \mathbf{\Lambda}_y$	loadings
$\mathbf{B}, \mathbf{\Gamma}$	structural paths
$\mathbf{\Phi}$	factor (co)variances
$\delta, \varepsilon, \mathbf{\Theta}$	errors; error (co)variances
ζ, ψ	disturbance, its variance

Which model do I need?



Two further branches: “does the effect pass through a mediator?” leads to a **mediation** model; repeated measurements over time lead to a **latent growth** model.

Mediation: whether and why



$$\text{indirect} = a b, \quad \text{total} = c' + a b$$

Two questions The *total* effect asks whether the programme works; the *indirect* effect asks *why*.

Why a bootstrap? $\hat{a}\hat{b}$ is not normal in small samples or when a path is weak. Resample, recompute $\hat{a}\hat{b}$, take percentiles (Preacher and Hayes, 2008).

A live mediation with a bootstrap interval

```
set.seed(99)                                # simulated data
N <- 800
training <- rnorm(N)
skill    <- 0.55 * training +
           rnorm(N, 0, sqrt(1 - 0.55^2))
turnover <- -0.50 * skill - 0.10 * training +
           rnorm(N, 0, 0.7)
med_dat <- data.frame(training, skill, turnover)
```

True values: $a = 0.55$, $b = -0.50$, $c' = -0.10$, so
 $ab = -0.275$.

```
med_model <- '
  skill    ~ a*training
  turnover ~ b*skill + cp*training
  indirect := a*b
  total    := cp + a*b
'
set.seed(2026)
fit_med <- sem(med_model, data = med_dat,
               se = "bootstrap",
               bootstrap = 1000)
```

Reading the mediation

```
pe <- parameterEstimates(fit_med,  
                          boot.ci.type = "perc")  
round(pe[pe$label != "",  
        c("est", "ci.lower", "ci.upper")], 3)
```

##		est	ci.lower	ci.upper
## 1		0.568	0.513	0.623
## 2		-0.517	-0.571	-0.458
## 3		-0.145	-0.204	-0.088
## 7		-0.293	-0.339	-0.251
## 8		-0.438	-0.496	-0.383

Rows: a , b , c' , indirect, total (95% percentile bootstrap CIs).

Interpretation Indirect effect $\hat{a}\hat{b} = -0.293$, 95% CI -0.339 to -0.251 : excludes 0, so training lowers turnover intention partly *through* skill. A direct effect $\hat{c}' = -0.145$ remains: partial mediation, about 67% of the total.

The model is saturated ($df = 0$): fit indices say nothing here.

PART 6

Good practice and reporting

Estimators and missing data

Estimator	Use when
ML	continuous, roughly normal indicators
MLR	non-normal: robust SEs and χ^2
WLSMV	ordinal items (few categories)
FIML	missing data (MAR): <code>missing = "fiml"</code>

Most common mistake Treating 3- or 5-point Likert items as continuous without checking.

Missing data Listwise deletion drops every incomplete case: wasteful, and biased unless data are missing completely at random. FIML uses all available information under MAR.

If stressed employees skip the workload items, deleting them makes the sample look less stressed.

Sample size and measurement invariance

Sample size: no single rule

- simple models: often $N \geq 200$
- 10, better 20, cases per free parameter
- weak loadings, ordinal data, non-normality, many groups: more
- plan important studies by Monte Carlo power analysis

Level	Also equal	Allows
configural	factor pattern	same structure
metric	loadings	compare relations
scalar	intercepts	compare latent means
strict	residual variances	strongest comparison

Test the ladder in order (`group.equal` in **lavaan**) before comparing groups on a latent construct.

Equivalent models and core assumptions

Good fit is not unique truth

Different diagrams can imply the same covariance matrix and fit identically: $X \rightarrow Y$ and $Y \rightarrow X$ cannot be told apart by fit. Theory, design and temporal order must decide.

An SEM assumes

- the theoretical structure is roughly right
- linear relationships, unless modelled otherwise
- an estimator that matches the data
- an adequate sample size
- no major omitted common cause

Troubleshooting

Symptom	Likely cause	Response
no convergence	poor start values, unidentified model, small N	simplify; rescale variables; check df
negative variance (Heywood case)	misspecification, outliers, near-duplicate items	inspect the structure; do not report as is
standardised loading above 1	empirical under-identification, collinearity	check the specification
huge standard errors	weak identification	more indicators; justified constraints
good fit, implausible signs fit improves after many edits	wrong theory or over-fitting capitalising on chance	rethink the diagram call it exploratory; cross-validate

Reading and reporting an SEM

Read the output in order

1. convergence
2. global fit: χ^2 , df, CFI, TLI, RMSEA, SRMR
3. measurement: loadings, residual variances
4. structure: paths, indirect effects
5. R^2 of endogenous variables

Report

- the diagram, N , data type, estimator, missing-data method
- $\chi^2(\text{df})$, p , CFI, TLI, RMSEA with CI, SRMR
- unstandardised and standardised estimates with SEs or CIs
- indirect effects with bootstrap CIs
- reliability and validity of each construct
- post-hoc changes, alternative models, limitations

Key take-aways

1. SEM reproduces the **observed covariance matrix** with a few theory-driven parameters.
2. It joins **measurement** (constructs $\rightarrow$ items) and **structure** (constructs $\rightarrow$ constructs), correcting for measurement error.
3. **Count** df before fitting: only over-identified models can be tested.
4. Judge fit by χ^2 **and CFI, TLI, RMSEA, SRMR together**; then check loadings, reliability and validity.
5. Test indirect effects with **bootstrap intervals**.
6. Arrows are assumptions: **report transparently**, including alternatives and post-hoc changes.

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Thank you

Questions and discussion